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Forecasting of immigration flows until 2025 for selected European countries using expert information

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1. Introduction

The aim of the report is to provide forecasts of immigration into seven European countries in the horizon of 2025, based on the quantitative data, as well as on the country-specific expert knowledge. The geographical coverage of the report corresponds to the countries participating in the research project "Mediterranean and Eastern European Countries as new immigration destinations in the European Union" (hereafter: IDEA), which comprise of Austria, Czech Republic, France, Greece, Hungary, Italy, Poland, Portugal and Spain. However, Greece has not been included in the forecasting exercise due to the lack of data on international migration flows and Spain due to unavailability of the expert information as of the date of the current report.

Forecasting migration is a very difficult research task, for the reasons including, though not limited to the following: (1) inherent randomness of the processes under study and their susceptibility to hardly predictable factors; (2) lack of coherent definitions of immigration across countries and time; (3) lack of comprehensive migration theories; and (4) lack of data or incomplete data, including short time series (details for example in Willekens, 1994, as well as Kupiszewski, 2002). On the other hand, migration forecasts with suitable uncertainty assessments are crucial for obtaining credible population predictions, especially for developed countries.

All the issues mentioned above call for the inclusion of expert knowledge in the forecasting exercise (Willekens, 1994). The natural methodology for handling the combination of subjective expertise and the data is the Bayesian approach. The expert judgments or opinions can be treated as prior knowledge represented by the prior probability distributions that is then combined with data reflected in the likelihood function by means of the Bayes theorem. Especially the informative priors, which take into account the hardly predictable nature of migration seem to reflect the uncertainty associated with the processes in question better than hardly- or non-informative distributions, which let the (flawed) data alone speak for themselves (Bijak, 2008b).

The variables forecasted in the current study are immigration inflows, both total and the ones from up to three most important sources of immigration (or citizenship groups, depending on data availability). As no harmonisation of migration data was envisaged in the project, the consequence is the lack of comparability of flows obtained for various countries. The adopted perspective is demographic, with the aim to ensure within-country consistency of the forecasted immigration volumes and population stocks, rather than enable between-country comparisons.

Apart from the current Introduction, the report is structured in six sections. Section 2 addresses the data issues specifically related to the current forecasting task. In Section 3, the forecasting methodology is presented, which is based on the Bayesian approach in statistics. Further, Section 3 focuses also on more technical issues related to the estimation of the parameters of the forecasting models, including the adopted numerical procedures.

The Bayesian perspective enables to formally combine quantitative data with the *a priori* country-specific expert knowledge. In the current study, the knowledge is obtained via a Delphi questionnaire survey, described in Section 4, and the elicitation procedure is explained in Section 5. Section 6 presents main forecast results obtained for particular countries, both for the global inflows, as well as for shares of immigration from various origins. Additionally, in Section 6, the impact of demo-economic variables on migration flows is assessed. Finally, Section 7 provides a summary of the results, as well as the main conclusions from the forecasting exercise, together with the most important recommendations both for forecast-makers and forecast-users.

The report also includes three appendices. Appendix A contains a tabular summary of the availability and completeness of data series used for the purpose of the forecasting exercise. Appendix B provides more detailed forecast results for the seven IDEA countries, in the form of graphs and tables. It also contains selected information on the assumptions with respect to the parameterisation of *a priori* distributions, and on the data support for particular forecasting models. A technical Appendix C lists sample programme code in the WinBUGS 3.0.3 software environment, which was used for the computations of forecasts.

2. Data sources and preparation

The current section discusses main issues concerning the data used in the preparation of the forecasts in the IDEA project. It addresses the definitions of the migration flows under consideration as well as the sources of data, their availability and reliability with respect to the totals and three most important countries – sources of immigrants. More detailed information concerning data issues and collection is available in the database report on Work Package 5 (IDEA Deliverable D5.2, Wiśniowski, Kupiszewska, Kupiszewski 2008).

The subject of the forecasting exercise were the total flows of immigrants to seven IDEA countries. The data lack international comparability due to the differences in the definitions of 'immigrants', such as the registration of the immigrants by country of previous residence, by citizenship, or by duration of stay. In some countries the definition of the immigrant changed, as for example in the Czech Republic in 2001. Table 1 presents the definitions of the immigration flows in IDEA countries used in the forecasting exercise.

The primary sources of the migration data used in the forecasting exercise include Eurostat, United Nations Statistics Division, national statistical offices and data provided in the Council of Europe's *Demographic Yearbooks*. The longest series are available for Spain (unused) and Italy, both starting in the early 1980s, the shortest series – 12 observations – is available for France. For Greece, no data are available. Some of the data required recalculation (totals and shares for Portugal, as well as shares for Austria[1]) The detailed information concerning the data sources is presented in the Appendix A.

[1] For Portugal the data were obtained from Eurostat and Statistics Portugal (INE). In 2006 the number of 62,332 immigrants who applied for the residence permit included also people, who arrived to Portugal in previous years and had been holders of stay permit. This number (31,605 people) was distributed over the years 1997–2005, as this was the period between the two data regularisations (see Sabino and Peixoto, 2008; credits go to João Peixoto for bringing these data issues to our attention). For Austria the recalculation included summing up the totals of immigrants from the ex-Yugoslav countries.

Table 1. Immigration flows definitions for IDEA countries

Country	Country of previous residence or citizenship	Duration of stay	Comments
Austria	Country of previous residence	3 months	Short- and long-term immigrants
Czech Republic	Country of previous residence	Permanent* / 3 months** / 1 year***	Permanent immigrants; since 2001 stay criterion
France	Citizenship	1 year	Foreigners with long-term settlement permits
Hungary	Citizenship	1 year	Registered long-term immigrants
Italy	Country of previous residence	Not specified** / 6 months***	Immigrants recorded by population registers
Poland	Country of previous residence	Permanent	Immigrants for permanent residence
Portugal	Country of previous residence	1 year	Persons with stay permits for a duration of at least 1 year
Spain (unused)	Country of previous residence	Not specified	Registered immigrants (for details see Kupiszewska and Nowok, 2008: 62)

* refers to nationals; ** refers to citizens of EEA countries; *** refers to non-EEA citizens

Source: Kupiszewska and Nowok (2008: 54–62), own elaboration.

The data for the vector autoregression models that assess the impact of the pre-selected economic and demographic indicators (see Section 3.3 for details) were taken respectively from the World Bank's *World Development Indicators* (WDI; World Bank, 2008), and from the Eurostat database (NewCronos, domain '*demo*'). Beside the total immigration flows, these variables are:

- Yearly percentage change of the PPP-adjusted GDP *per capita* (constant 2000 US$),
- Total unemployment rate (% of the total labor force),
- Annual natural population growth: births minus deaths (% of the 1st January population),
- Population in the age 15–64 years (% of the total population as of 1st January).

The demographic series begin and end in the same years as the ones for immigration totals, while the economic series finish in 2006 at the latest. For 2006 the unemployment rates were not available from the WDI, instead, the UN Economic Commission for Europe data were used[2], adjusted by adding the differences between the UNECE and World Bank data observed in 2005.

3. Methodological framework for immigration forecasting

3.1. Introductory notes: forecasting and probabilities

The aim of this section is to provide insights into the methodology of forecasting international migration flows for the period 2007–2025 for the countries relevant to the IDEA project. The

[2] Source: http://w3.unece.org/pxweb/DATABASE/STAT/20-ME/3-MELF/3-MELF.asp, variable *"Unemployment Rate by Country and Year"* (accessed on 5 November 2008).

current introduction covers basic terminological issues concerning forecasts and forecasting, together with a brief synopsis of various topics related to the probability concept.

To start with, after Keilman (1990: 7), let *forecast* be defined as an *unconditional* result of the process, in which "based on current scientific insights, a forecaster gives his *best*[3] guess of what the future […] will be." Contrary to common perception, the primary aim of socio-economic forecasting is *not* to predict the future with a 100 percent accuracy, but rather to provide input to guide the political decision making process (Duchêne and Wanner, 1999). In such way, through interactions between forecasting and decision making, social science "no longer merely investigates the world; it creates the world it is investigating." (Boulding, 1969: 3).

In this context, the role of a forecaster is to ensure that the 'best guess' about the future is well-informed, follows the methodological state-of-the-art, and takes into account possibly all relevant aspects of the phenomenon under study, at least to the extent available at the time of preparing the forecast. By no means should forecasters refrain from preparing forecasts using a convenient excuse that they would not come true anyway. On the other hand, also forecast users should not expect the impossible: point forecasts of most socio-economic variables are almost certain *not* to be fulfilled in terms of the exact values.

For the above-mentioned reasons, the key issue in the forecasting process becomes not to offer an (improbable) point estimate of the future values of the variables under study, but rather to provide a reliable assessment of the related uncertainty span, ideally, in a coherent and quantifiable manner. Appropriate tools to achieve this aim are offered by the probability theory and statistical inference. As noted by Dawid (1984: 278), "one of the major purposes of statistical analysis is to make forecasts about the future [and] to offer suitable measures of uncertainty associated with unknown events or quantities."

Formally, uncertainty is best described in terms of probability distributions, which are a mathematical representation of the features of unknown (random) quantities. The concept of probability as a measure of uncertainty can be either defined in relation to the frequency of events under study (the classical approach), or as a measure of belief of a researcher in the occurrence of these events (the subjectivist approach). In either case, probability is bound to fulfil several propositions (axioms), notably, to be limited to the range of values between zero for (almost) impossible events and one for (almost) certain events.

Examples of some basic probability distributions for continuous variables are illustrated in Figure 1 in terms of *density functions*[4]. Two upper graphs present the Normal and Uniform distributions, which can be used, among others, for expressing our uncertainty with respect to the expected location (or central tendency) of the phenomenon under study. In the case of the the Normal distribution, we expect the variable to be more likely located around some central value (peak) than elsewhere, while the Uniform distribution reflects equal chances of its emergence anywhere within a pre-defined interval.

Two lower graphs show different distributions from the Gamma family, which are defined only for positive numbers and can be used for example to express uncertainty on the precision of the phenomenon, in other words, for assessing how precise are we with respect to our

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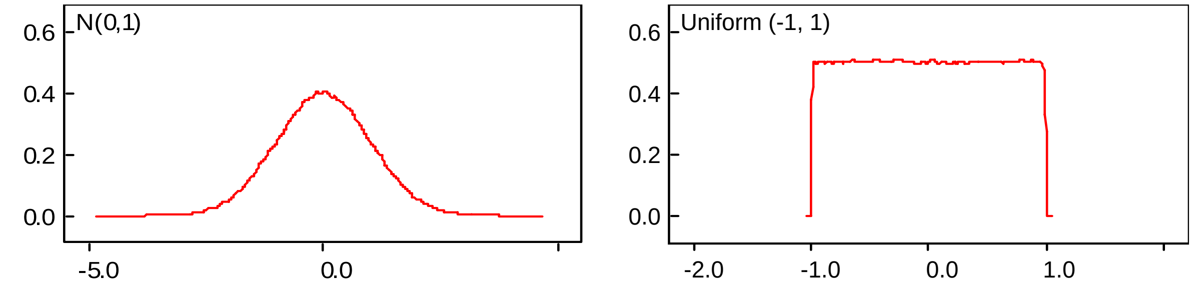
[3] Emphasis ours.

[4] Formally, the probability that the variable in question will take values between *a* and *b* is equal to the area (integral) under the curve depicting the density function, limited to the [*a*, *b*] interval on the horizontal axis.

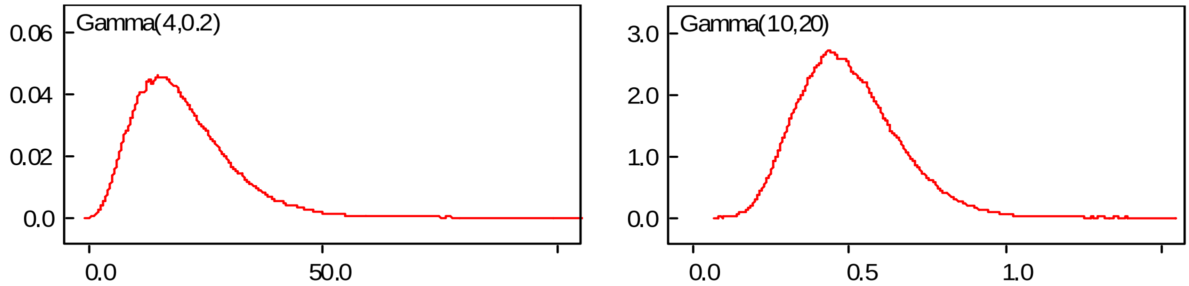
estimations or forecasts. Technically, precision τ is defined here as the inverse of the variance σ^2 of a random variable under study, $\tau = \sigma^{-2}$. Higher precision (as illustrated in the left-panel graph) corresponds in this example to smaller uncertainty than lower precision (right panel).

Figure 1. Examples of density functions of some basic probability distributions

Location – examples of Normal and Uniform distributions:



Precision – examples of two different Gamma distributions:



Source: own elaboration in WinBUGS 1.4.

3.2. General forecasting framework applied in the current study

There is a rather clear agreement in the mainstream of population forecasting that the future belongs to probabilistic predictions, due to their ability to quantify uncertainty in a proper and coherent manner (Lutz and Goldstein, 2004: 3–4)[5]. Migration forecasts are here by no means an exception, regardless of the difficulties associated with the task, that is, of high predictive uncertainty associated with the very nature of the migratory processes. In order to accommodate the specific features of migration in a proper fashion, the proposed forecasting methodology extends the traditional statistical methods with the aim to formally incorporate judgemental elements in the forecasts, following the suggestions of Willekens (1994).

Among the forecasting methods that incorporate uncertainty in a formal, quantitative manner, based on the notion of probability, several possibilities can be considered. First, the most widely-used is the time series analysis based on the classical (*frequentist* or *sampling-theory*) paradigm of statistical inference, which is exclusively based on the data at hand, and the model parameters are treated as fixed, albeit unknown.

In turn, in the Bayesian paradigm of statistical inference, based on the Bayes' Theorem (Bayes, 1763), the sample information is used to transform the prior knowledge (*a priori*) of

[5] The uncertainty issue has been already acknowledged and discussed on the international level, for example during the *Joint Eurostat – UN ECE Work Sessions on Demographic Projections*, two recent ones having been held in Vienna, 21–23 September 2005 (http://circa.europa.eu/irc/dsis/jointestatunece/info/data) and in Bucharest, 10–12 October 2007 (http://www.unece.org/stats/documents/2007.10.projections.htm), where the expert panels agreed on the necessity to include uncertainty assessments in population forecasts.

the researcher with regard to the phenomenon under study, into the posterior knowledge (*a posteriori*). The former may reflect the subjective opinion (belief, intuition) of the expert on the subject, without taking observations into account, while the latter is conditional on the sample data. The Bayesian statistics infers on the unknown parameters of the model describing the phenomenon (θ), treated here as random quantities, conditionally on the statistical information (x), unlike in the traditional sampling-theory statistical methods, for example in the Neyman-Pearson theory of hypothesis testing. One element that is shared by both approaches is the *likelihood* of the data, that is, the probability that a given sample (the data at hand) was generated by a model with parameters θ. The scheme of Bayesian inference and the Bayes Theorem are presented in Box 1.

Box 1. The Bayes Theorem and the Bayesian statistical inference

posterior knowledge	=	prior knowledge (*experts*)	·	likelihood (*data*)
$p(\theta \mid x)$	=	$p(\theta)$	·	$p(x \mid \theta) / p(x)$

An important issue is the selection of the prior probability distribution of the model parameters, $p(\theta)$, reflecting either the knowledge of the researcher, or lack thereof, as in the case of *non-informative distributions*. The role of the latter is to formally reflect the researchers' ignorance with respect to the parameters in question: in some instances, Uniform distributions or other, although rather diffuse ones, can be used for this purpose. Selection of an informative prior distribution is usually supported by the expert judgement. An analysis of robustness of the results against changes in the prior distribution is thus an important element of the Bayesian inference. A natural outcome of the analysis is the posterior distribution $p(\theta \mid x)$. Probability in the Bayesian statistics is subjective, independent of the frequency of events under study.

The posterior knowledge on the parameters of a forecasting model subsequently serves as an input to produce forecasts, here, the whole *predictive distributions* concerning the future values of migration-related variables. Such distributions, which describe the uncertainty of forecasts in a formal and coherent way, can be summarised by their point characteristics (means, medians, quantiles, etc.), or credible regions, analogous to confidence regions in the sampling-theory statistics. In migration forecasting, the Bayesian approach has been so far successfully applied in a handful of studies (Gorbey *et al.*, 1999; Brücker and Siliverstovs, 2005; Bijak, 2008a and 2008b), where it proved to yield similar *ex-post* errors, yet more realistic predictive intervals than the classical approach. Bayesian inference can be seen as an alternative to 'expert-based' forecasting proposed by Lutz *et al.* (2004), which relies on the expert judgement, although making use neither from the Bayesian inferential mechanism, nor from full information from the data sample. A detailed literature survey on migration forecasting methods is covered in Bijak (2008a and 2008b).

There are several important arguments for using the Bayesian approach in migration context. Firstly, in many cases the series of data may be too short to allow for a meaningful classical inference. Secondly, in the case of many migration flows traditional methods can underestimate the uncertainty of forecasts (*idem*). For these reasons, the proposed forecasting methodology is based on Bayesian statistics, where expert knowledge on the processes under study can play an important role next to the observed data samples. Here, the role of judgement in the forecasting process becomes crucial, in order to accommodate qualitative scenarios on country-specific migration developments. As described further in Sections 4 and 5, the relevant judgements have been elicited from the country experts, by means of an e-mail questionnaire[6]. Subsequently, both the expert knowledge and quantitative data have been combined within the forecasting models and ultimately yielded predictions presented in Section 6 of the report.

3.3. Specification of the forecasting models

3.3.1. Forecasts of total migration inflows

With respect to model specification, several aspects of the forecasting exercise have been considered. Firstly, the basic analysis has been limited to total immigration, and up to three most important immigration flows (subject to data availability), which for the sake of consistency have been defined in terms of shares rather than absolute values. The directions of inflows (or respective citizenships, as indicated in Table 1) have been indicated by the country experts in the Delphi survey described in Section 5, and the 'rest of the world' category was obtained as a residual value. The main forecasted variable is a log-transformed migration volume rather than any of the related intensity measures (rates or ratios), for it seemed much more natural to elicit expert knowledge on the absolute size of flows rather than any relative indicators.

To start with, the basic model space covers four models M_i, denoted as $M_1 – M_4$. Two of them (M_1 and M_3) are autoregressive models of the first order, AR(1), additionally containing a deterministic trend, while the remaining ones (M_2 and M_4) are random walk models with drift (RW). Out of each pair, the first model is characterised by constant variability (CV), while the second one by conditional variance changing according to the simplest 'stochastic volatility' scheme (SV). The model equations are listed below (cf. Greene, 2000, *passim*):

M_1: $\ln(m_t) = c_1 + \gamma_1 \cdot trend_t + \phi_1 \cdot \ln(m_{t-1}) + \varepsilon_{1t}$, and $\varepsilon_{1t} \sim$ iid $N(0, \sigma_1^2)$; (1a)

M_2: $\ln(m_t) = c_2 + \ln(m_{t-1}) + \varepsilon_{2t}$, and $\varepsilon_{1t} \sim$ iid $N(0, \sigma_2^2)$; (1b)

M_3: $\ln(m_t) = c_3 + \gamma_3 \cdot trend_t + \phi_3 \cdot \ln(m_{t-1}) + \varepsilon_{3t}$, and $\varepsilon_{1t} \sim N(0, \sigma_{3t}^2)$, where

$\ln(\sigma_{3t}^2) = K_3 + \psi_3 \cdot \ln(m_{t-1}) + \xi_{3t}$, and $\xi_{3t} \sim$ iid $N(0, \upsilon_3^2)$; (1c)

M_4: $\ln(m_t) = c_4 + \ln(m_{t-1}) + \varepsilon_{4t}$, and $\varepsilon_{1t} \sim N(0, \sigma_{4t}^2)$, where:

$\ln(\sigma_{4t}^2) = K_4 + \psi_4 \cdot \ln(m_{t-1}) + \xi_{4t}$, and $\xi_{4t} \sim$ iid $N(0, \upsilon_4^2)$. (1d)

In (1a) – (1d), m_t universally denotes migration inflow (according to a country-specific definition), which is log-transformed in order to ensure positive migration flows and asymmetry of predictive distributions (heavier upper tails). Further, c_i are constants, t is the time index, $trend_t$ is a country-specific trend function, and ε_{it} are random (noise) terms, assumed to follow Normal distributions[7] with mean 0 and variance σ_{it}^2, $N(0, \sigma_{it}^2)$. The abbreviation 'iid' denotes that the random variables concerned are independent and identically distributed. In all cases, the model-specific indices i of parameters $\theta \in \{c, \gamma, \phi, K, \psi, \sigma, \upsilon\}$ denote in fact not different parameters, but merely depict a given parameter in the i-th model, $\theta_i = (\theta|M_i)$. Further particulars of the models for specific countries are presented in Section 3.5, devoted to computational issues.

[6] For a discussion of various problems related to the elicitation of expert judgement within the Bayesian context, involving selected issues related to the psychology of the elicitation process, see Kadane and Wolfson (1998).

[7] Technically, it is worth noting that for the log-transformed variables, $\ln(m_t)$, the predictive distributions yielded by a Normal likelihood and *a priori* Gamma-distributed precision (inverse variance), in simple models follow the Student-*t* distribution (e.g., in linear regression models, which, however, need not be exactly the case in non-linear models). Thus, they have heavier tails than in the Normal distribution, especially for such small samples as presented here.

The ultimate forecasting models have been selected from the above-listed ones on the basis of the *posterior odds* criterion. The general idea behind such model selection is also Bayesian, and consists in defining prior probabilities over the model space, $p(M_i)$, for models M_i, $i = 1,\ldots,4$, assuming that the models are mutually non-nested (to fulfil this assumption, $\phi_1 \neq 1$ and $\phi_3 \neq 1$ need additionally hold for models M_1 and M_3). Subsequently, the prior probabilities of specific models are combined with the marginal densities of the observations vector from particular models M_i. As outcome, posterior model probabilities $p(M_i|x)$ are yielded by the Bayes rule presented in Box 1, which in this case has a form (e.g., Osiewalski, 2001: 21):

$$p(M_i|x) = p(M_i) \cdot p(x|M_i) \,/\, p(x); \qquad (2)$$

For the forecasting, the model with the highest value of the posterior probability $p(M_i|x)$ is eventually selected. In such cases, when more than one model has a relatively high posterior probability (arbitrarily assumed as over 0.05), the ultimate forecasts can be obtained as weighted averages from the predictive distributions[8]. Technically speaking, such averaged distributions are discrete mixtures of predictive distributions yielded by particular models, with mixing obtained using a categorical distribution defined by the posterior probabilities (2), treated here as 'weights'.

3.3.2. Forecasts of origin-specific immigration shares

The second modelling task consists in predicting the main directions of inflows, as stated before, defined in terms of shares rather than absolute numbers, to ensure consistency of the results with global forecasts. Shares are by definition constrained to the [0, 1] interval and have to add up to unity (100%). Therefore, in order to facilitate the modelling by by-passing these restrictions, the original variables were re-calculated using a multinomial logit transformation, following one of the possibilities of the compositional data analysis (cf. Theil, 1969).

Letting $m_t^{(1)}$, $m_t^{(2)}$ and $m_t^{(3)}$ denote immigration for the three most important directions (or citizenships) of inflow, and $m_t^{(0)}$ the one for the 'rest of the World' category, the following identity holds: $m_t^{(1)} + m_t^{(2)} + m_t^{(3)} + m_t^{(0)} = m_t$. Denoting the respective shares by $\alpha_t^{(i)} = m_t^{(i)}/m_t$, this is equivalent to $\alpha_t^{(1)} + \alpha_t^{(2)} + \alpha_t^{(3)} + \alpha_t^{(0)} = 1$. The shares, taken relatively to the 'rest of the World' category, are subsequently log-transformed into new variables $z_t^{(i)}$, according to the formula:

$$z_t^{(i)} = \ln\left(\alpha_t^{(i)} / \alpha_t^{(0)}\right), \text{ for } i \in \{1, 2, 3\}. \qquad (3a)$$

Such transformation ensures that the variables $z_t^{(i)}$ are defined over the whole space of real numbers and that the modelled and forecasted shares sum up to the totals after back-calculation:

$$\alpha_t^{(i)} = \exp(z_t^{(i)}) \,/\, (1 + \exp(z_t^{(1)}) + \exp(z_t^{(2)}) + \exp(z_t^{(3)})), \text{ for } i \in \{1, 2, 3\}. \qquad (3b)$$

The transformed variables have been modeled and forecasted jointly, in order to capture the possible interaction effects between particular directions of inflow, both instantaneous and with a time-lag of one year. The model is a (maximally) three-dimensional vector

[8] More on Bayesian model selection and forecast averaging (also known as 'inference pooling') can be found for example in Hoeting *et al.* (1999) and Osiewalski (2001).

autoregression of order 1, VAR(1), where the joint vector of all variables under study, $\mathbf{x_t}$ = $[z_t^{(1)}\ z_t^{(2)}\ z_t^{(3)}]'$ is assumed to be subject to the following multivariate process (cf. Greene, 2003):

$$\mathbf{x_t} = \mathbf{c} + \phi\ \mathbf{x_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t}, \text{ and } \boldsymbol{\varepsilon}_\mathbf{t} \sim \text{iid } \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}). \quad (4)$$

The vector **c** includes constants, as well as deterministic trends, if they were indicated by the experts, ϕ is a square matrix of model parameters, and $\boldsymbol{\varepsilon}_\mathbf{t}$ is a multidimensional random term, here assumed to follow a multivariate Normal distribution **N** with mean vector **0** and the covariance matrix **Σ**. As regards the dimensions of the model, in the current study the analysis involves three countries of origin or citizenship, apart from Poland, for which the experts indicated only two outstanding directions of expected future migration inflows.

3.3.3. Impact of demographic and economic variables on immigration

The third modelling issue concerns the use of migration theories and, in result, additional explanatory variables in the forecasting models. As the limitations of migration theories are well-known (see e.g. Arango, 2000), the analysis was constrained to migration determinants known to be important for migration flows in Europe (cf. Jennissen, 2004). In the current study, four such determinants have been chosen, two of them being economic and two demographic. The former group includes economic growth, denoted by x_1 (measured by the annual rates of growth of the PPP-adjusted GDP per capita), as well as unemployment rates (x_2). The latter group consists in the rate of natural population growth (x_3), defined as births less deaths relative to the population size, treated here as exogenous, as well as in the share of the productive age group (age 15–64) in the total population (x_4). Unfortunately, due to the shortness of the time series, forecasting models could not include historical analogies to other countries, which would refer to the concept of various stages in migratory processes observed in more 'mature' migration countries, as demonstrated especially in the IDEA report on Austria (Fassmann and Reeger, 2008).

In this part of the study, VAR(1) models are examined, with modelled vectors comprised either of the economic variables $\mathbf{x_t} = [\ln(m_t)\ x_{1t}\ x_{2t}]'$ or demographic variables $\mathbf{x_t} = [\ln(m_t)\ x_{3t}\ x_{4t}]'$. The models have therefore a form akin to (4). The separate treatment of both models is not only due to the different nature of the respective variables in question, but also to the fact that including interrelations between the economic and demographic parts far exceeds both the scope of the current analysis, as well as the amount of available expert information. In the absence thereof, a joint treatment of the two parts would likely result in an artificial inflation of model uncertainty, embodied through the respective matrix parameters **A** and **Σ**.

The impact of particular determinants on migration, both instantaneous, as well as with a time-lag of one year, is subsequently assessed using Lindley's test, here being a Bayesian equivalent of classical Wald's tests for significance of restrictions (for details on the numerical procedure, see Bijak, 2008b: 88–93). Let $\boldsymbol{\beta}$ denote the test statistic. Firstly, $\boldsymbol{\beta}$ can be a vector of the respective parameters of the matrix **A** (a_{12} and a_{13}), treated either separately or jointly, indicating the **lagged** impact of demographic or economic variables on migration. Secondly, $\boldsymbol{\beta}$ can contain the parameters of the regression of migration, $\ln(m_t)$, on either economic (x_{1t} and x_{2t}) or demographic (x_{3t} and x_{4t}) covariates. Such parameters are derived from the covariance matrix **Σ** and pertain to the **instantaneous** impact. Thus, altogether, six combinations of various $\boldsymbol{\beta}$ can be considered for each of the models, including cases when particular variables are treated jointly as well as separately, and the analysed impact can be both immediate and lagged.

In a general case, under the null hypothesis of $\boldsymbol{\beta} = \mathbf{0}$, indicating no particular impact of other variables on migration, the test statistic has a form (Greene, 2000: 153–156, after *idem*: 92)[9]:

$$\psi(\boldsymbol{\beta}) = [\boldsymbol{\beta} - E(\boldsymbol{\beta} \mid \mathbf{x})]'[Var(\boldsymbol{\beta} \mid \mathbf{x})]^{-1}[\boldsymbol{\beta} - E(\boldsymbol{\beta} \mid \mathbf{x})], \qquad (5)$$

The critical values of the one-tailed test are derived numerically. Large values of $\psi(\boldsymbol{\beta})$ lead to the rejection of the null hypothesis and thus indicate a significant impact of particular covariates (or their combinations) on migration. The test results subsequently serve as guidance for reducing the initial models according to the 'from general to specific' principle, by removing the variables that have no impact on migration (see Section 6).

It has to be noted (cf. Bijak, 2008b) that in such multivariate models as VARs, the forecasters and forecast users should generally expect very high predictive uncertainty, stemming from three sources: (1) randomness of migratory processes as such; (2) uncertainty of the covariates; and (3) uncertainty of their mutual interrelations, embodied in the parameters of the forecasting model (matrices **A** and **Σ**). In the light of the limitations of the use of theories in forecasting, the main purpose of applying multivariate models in prediction-making is their ability to produce coherent 'what-if' scenarios. A scenario analysis, if relevant e.g. for policy considerations, can be easily obtained at a later stage by conditioning the forecasts obtained from the model on *given* trajectories of the covariates.

The key advantages of such an approach over the 'plain' scenario setting, traditionally applied to migration projections, are twofold. Firstly, the traditional scenario analysis does not provide any indications as to the uncertainty of future migration flows: there is no information, what are the expected chances that the forecasted variable will fall between various scenarios (Lutz *et al.*, 2004: 19). Secondly, the construction of scenarios is usually arbitrary and not supported by the in depth analysis and a proper quantification of links between the explanatory part ('if') and migration developments ('then').

As illustration, the current study presents the conditional forecasts from VAR models with demographic covariates, wherever their impact on migration was found significant, that is, for all countries except Portugal. The reasons for selecting demographic models were twofold: (1) in the case of economic models, hardly any impact was detected, and (2) it was much more reasonable to think of long-term scenarios in the case of demographic variables. Here, the scenarios of the natural population growth rate (x_3) and the share of the productive age group (x_4) were taken from the main variant of the recent population projections of Eurostat (EuroPop 2008)[10]. Details concerning models and their results for particular countries are provided in Section 6.4.

[9] We are grateful to Prof. Jacek Osiewalski for suggestions on the Bayesian variant of Wald's test.

[10] Source: NewCronos database, domain '*proj*' (population projections), subdomain "EUROPOP2008 – Convergence scenario, national level", tables for Convergence year 2150. Data available from http://epp.eurostat.ec.europa.eu (accessed on 3 November 2008).

3.4. Country-specific modelling issues

During the modelling process several country-specific issues had to be solved, which are described in the current part of the report. These issues concern two main areas: (1) deterministic trends, which had to be incorporated into the models (both for totals and for shares) in such a way so as to remain coherent with the information elicited from the experts, and (2) dummy variables used to handle some specific characteristics of the data.

In general, the trends were incorporated in the autoregressive (AR) models and models for shares, but not in the random walk (RW) ones. It was motivated by the characteristic of the random walk process. For all countries but Poland the trend for immigration flows indicated by the experts was logarithmic, whereas in the Polish case it followed a logistic pattern.

In the case of the shares models, it should be highlighted, that the explicitly modelled variables were the transformations of the original shares, as shown by equations (3a) and (3b), and hence the interpretation of the trends, as well as the parameter values is heavily hampered.

For **Austria**, there were 12 observations on the totals (for details on data see Section 2 and Appendix A) and 18 periods for prediction. The trend incorporated in the model for totals was logarithmic. Note, that in order to reflect the logarithmic tendency, the trend included in the model for logarithms of inflows should be double-logarithmic, i.e. the variable $trend_t$=ln(ln(s)),
s = 2, 3, … , 30 was used.

In the case of the shares model, the trend was introduced for Turkey and for ex-Yugoslav countries, as it was indicated by the experts (in question 13 of the Delphi survey, for details see Section 5.2) that these shares will be increasing in the period under consideration. That was handled by a logarithmic-type trends given by equation (6). As it was reasonable not to assume a trend in the sample period, the first part of the trend variable was constant, what, in the end, allowed the desired shares to rise in the forecasts.

$$trend_t = \ln(\ln(s)), \qquad s = \begin{cases} e & for \quad t = 2, 3, \ldots, 12 \\ 3, 4, \ldots, 20 & for \quad t = 13, \ldots, 30 \end{cases}. \tag{6}$$

In the case of the **Czech Republic**, there were 15 observations available and 18 prediction periods. The trend included in the model for totals was based on the indications given by some of the experts, such that the immigration flows would increase but at the end of the forecasting horizon they would start an ever-slower decreasing (likely after 2013–2015). The construction of the trend function was also aimed at handling some technical issues, such as the problem with explosiveness of the forecasts. Hence, the trend was again $trend_t$=ln(ln(s)), where:

$$s = \begin{cases} e & for \quad t = 2,3,\ldots,9 \\ 3, 4, \ldots, 14 & for \quad t = 10, \ldots, 21 \\ 14, 13, 13, \ldots, 8 & for \quad t = 22, \ldots, 33 \end{cases}. \tag{7}$$

It has to be noted that such trend already accommodated the change of definition of immigration used in the Czech Republic (as described in Section 2) in the autoregressive models M_1 and M_3. Besides, the trend (7) was defined by including additional information from the comments of two Czech experts, indicating first an increase and then a decrease of total immigration levels. In this particular instance the additional 'descriptive' expert judgement proved to be very helpful.

For the model of shares, the double logarithmic-trend was included only in the equation describing the shares of Vietnamese immigrants, as was stated by the experts. In the case of Slovakia and Ukraine the experts either indicated the stability of shares or were divergent in their opinions. The trend and the motivation behind the use of it were similar to the ones used for Austria, as given in equation (8).

$$trend_t = \ln(\ln(s)), \qquad s = \begin{cases} e & for \quad t = 2, 3, \ldots, 12 \\ 3, 4, \ldots, 23 & for \quad t = 13, \ldots, 33 \end{cases}. \tag{8}$$

In random walk models for totals an additional dummy variable was included. It handled the change of the definition of an immigrant at the turn of the centuries and was of the form:

$$dummy_t = \begin{cases} 1 & for \quad t = 2, 3, \ldots, 9 \\ 0 & otherwise \end{cases}. \tag{9}$$

In the model for shares it was assumed that the change in definition did not affect the shares of the directions thus there was no dummy in there.

For **France**, the series available included 12 observations, whereas the forecasting period 20 observations. The trend introduced in the model was logarithmic (thus linear for totals), $trend_t = \ln(s)$, s = 1, 3, … , 32 as was indicated by the experts[11]. In the case of model for shares (12 observations), we incorporated the double-logarithmic trend of the form:

$$trend_t = \ln(\ln(s)), \qquad s = \begin{cases} e & for \quad t = 2, 3, \ldots, 12 \\ 3, 4, \ldots, 22 & for \quad t = 13, \ldots, 32 \end{cases}, \tag{10}$$

for Turkey and China, and no trend for Morocco (according to the experts' indication).

As far as the models for **Hungary** are concerned, there were 17 observations in the sample and 19 forecasting periods. The trend in the totals models was, again, double-logarithmic, in the form of $trend_t = \ln(\ln(s))$, s =3, 4, … , 38.

In the shares' model (here, only 15 observations were available), the trend was double-logarithmic and included in all three equations. For Ukraine and Serbia an increasing tendency was suggested by the experts, while in the case of Romania (the first equation) the trend was included in order to ensure the indicated stability of shares, which had the form:

$$trend_t = \ln(\ln(s)), \qquad s = \begin{cases} e & for \quad t = 2, 3, \ldots, 15 \\ 3, 4, \ldots, 21 & for \quad t = 16, \ldots, 34 \end{cases}. \tag{11}$$

[11] In fact, 40% of the experts indicated no trend and 40% – a linear trend. We decided to use the latter one, as it seemed to be more reasonable in order to fit the data at hand. Moreover, the sensitivity analysis performed with respect to models with and without trend showed that it virtually did not affect the results on the model selection – in both cases the best model was random walk with probability greater than 0.99 (for details see Section 6.1).

The sample data available for **Italy** contained 25 observations (though in the case of shares, only 10 data periods were available). The forecasting horizon comprised of 20 periods. Here again the trend had the same form as in Hungary and Austria, namely $trend_t = \ln(\ln(s))$, $t=3, \ldots, 47$. In the case of the shares model, no trend was introduced. The reasons were the very short time series, as well as rather vague information from the experts concerning the tendencies of the future behaviour of the inflows from the main directions.

For **Poland** there were 18 observations in the sample and the forecasting horizon was 18. The experts' opinions suggested strongly the logistic characteristic of the future immigration flows. Logistic trend was used in the AR models. The general formula for the logistic function is:

$$y_t = \frac{\alpha}{1+\beta \exp(-\gamma t)}, \qquad t = 2,3,\ldots,36. \tag{12}$$

Parameter α handles the asymptotic value of the future immigration flows, parameters β and γ are responsible for the curvature and the inflection point. After taking the logarithm of both sides we obtain:

$$\ln y_t = \ln \alpha - \ln (1+\beta \exp (-\gamma t)). \tag{13}$$

The second part of the right-hand side of equation (13) was transformed using the formula for the inflection point: $t^* =$ ln□/□̰̃Thus the trend in models M_1 and M_3 ((1a) and (1c)) was:

$$trend_t \tilde{=} \ln (1+\exp(\gamma(t^{*}\,\tilde{}\; t))), \qquad t = 2, 3, \ldots, 36, \tag{14}$$

with only one parameter γ to be the subject of inference. The $\ln \alpha$ in the equation (13) was set to be equal to the model-specific constant, c.

In the case of the model for shares, only two directions were chosen on the basis of the experts' answers: Ukraine and the United Kingdom (the latter being predominantly a source of returning migrants). The specific trend was incorporated in both equations, which was underlain by the experts opinions indicating the initial increase and then the stabilisation or decrease of both shares. Thus, in order to reflect a turning point around 2019 in the behaviour of shares, the trend was of the form: $trend_t = s$, where $s = 2, \ldots 22, 23, 22, 21, \ldots, 10$.

The data for **Portugal** contained 15 observations and 19 periods for forecasting. The trend included in the totals models was double-logarithmic (again, according to the elicited expert judgment), taking the form of $trend_t = \ln(\ln(s))$, $s= 3, 4, \ldots, 34$.

The model for shares did not contain the trend. In both models, however, a dummy was introduced. It handled the mostly unwelcome variation of the data in the last period of the sample (2006), the great value of which was resulting from a recent regularization process of the migration data in Portugal (see the first footnote in Section 2). Thus, by the dummy of the form

$$dummy_t = \begin{cases} 1 & for\ t = 15 \\ 0 & otherwise \end{cases}, \tag{15}$$

the last observation was deprived of influence on the forecasted tendencies.

3.5. Numerical issues

The current section presents selected numerical and technical details related to the estimation of the models and computation of the forecasts.

In order to obtain the posterior characteristics of the parameters, as well as the predictive densities (see in Section 3.2) for totals and shares models, a Gibbs sampling algorithm was used, which is a version of the Markov Chain Monte Carlo (MCMC) simulation technique (for details see e.g. Casella and George, 1992: 168, Osiewalski, 2001: 39). The algorithm consists in iterative sampling from the conditional distributions for the parameters, starting from some pre-defined initial values. Usually, a number of first samples is discarded (a so-called *burn-in* phase) so as to assure the convergence to the posterior distribution and eliminate the starting point effects. The rest of the sample is then used to compute the characteristics of the posterior distribution (from now on referred to as the *posterior sample*). Having obtained the posterior densities of the parameters, the samples from predictive distributions are generated. This part of the computations comprised the so-called phase #1.

For the problem of model selection the algorithm of Carlin and Chib (1995, see also a discussion in Bijak, 2008b) was applied. This technique allows to accommodate the computation of the posterior probabilities on the model space (see Section 3.3) within the Gibbs sampling procedure. The method consists of an iterative sampling from full conditional distributions for model-specific parameters (the characteristics of which are obtained in phase #1), as well as the model index. The Carlin-Chib procedure constituted the second phase of the computations. In the case of Hungary, Poland and Portugal the algorithm for averaging the forecasts from different models was applied, which comprised the computational phase #3.
The algorithm was implemented within the WinBUGS 3.0.3 (also called OpenBUGS) software, developed by Spiegelhalter *et al.* (2007). The sample code for the algorithms is presented in Appendix C. For verification of the convergence, the heuristic method of observing the quantiles, as well as the autocorrelations of the samples was applied. The number of samples differed across models and countries.

In the case of models for totals, in phase #1, the usual burn-in sample size was 50,000 and the number of samples from assumed posterior distribution was 150,000 (in the case of Poland the burn-in sample size was 850,000 due to the non-linearity with respect to the parameter was involved in the model). In order to reduce the autocorrelation of the samples for some parameters (usually these were the constants, trend and autoregressive parameters in the AR models), the so-called *k-thinning* was applied. This means that only every k^{th} iteration from each simulation was selected, which ultimately contributed to the calculation of the posterior density characteristics. The k was chosen depending on the shape of the autocorrelation functions and ranged from 10 to 25 (except for Poland, where thinning was not used). The sample sizes were enough to ensure the reasonable stability of the quantiles of the parameters, as well as of the predictive densities.

In the Carlin-Chib phase (#2) the burn-in and posterior samples were of the same size as in the phase 1 except for Poland, where they equalled 100,000 and 900,000 iterations respectively. For the averaging of the forecasts (phase #3), in the models for Hungary and Portugal the sample sizes were of the same size as in phases #1 and #2, while in the case of Poland, it was 750,000 and 250,000. The choice of k was the same in all three phases in all countries.

For the shares models, the burn-in sample sizes totalled usually 300,000 and the posterior 200,000 iterations (in the case of Poland it was 850,000 and 150,000, while for the Czech Republic – 800,000 and 200,000). No thinning was used in any of the countries. Again the quantiles showed the great level of stability of the densities. No thinning was required, as the autocorrelation of the samples for the parameters' densities was acceptable. From the posterior characteristics (medians) of the predictive distributions for the transformed variables, z, the predicted shares, α, were unravelled according to (3b). Because of the features of the logit transformation used (see Section 3.3), the calculation of the inter-quartile range and hence the measure of the uncertainty concerning the shares was not possible. This measure is available for the transformed variables z, although it is not interpretable and may not be consistent with results in terms of the shares, α.

The MC errors[12] in most of the models were reasonably small as far as the models' parameters are concerned. However, in some forecasts produced by some of the models, the MC error used do grow together with the forecasting horizon (e.g. in the case of AR–SV model for Poland's totals), which was the result of the presence of outliers sampled in some iterations. This fact served as a motivation behind making all of the inference in all described models basing on the location parameters (i.e. the median and quantiles of the distributions), which are free of the outlier problem, rather than the mean and the standard error.

As far as the vector autoregression models for the impact of the economic and demographic variables assessment are concerned, the sampling procedure comprised of 3 phases. In phases #1 and #2 the estimators of the expected values and variances *a posteriori* for the Lindley's test statistics were calculated. The outcome of phase #3 contained the table of the critical values for the test. The usual sample in each phase consisted of 150,000 iterations and the burn-in of 50,000. In the case of economic models thinning of order 25 was usually used, whereas for demographic models the thinning parameter ranged from 50 to 100. Additionally, in order to reduce the strong autocorrelation of the MC samples, the so-called *over-relaxing* was applied, i.e. an algorithm that generates multiple samples at each iteration and then selects the one that is negatively correlated with the preceding one (for details see Spiegelhalter *et al.*, 2007). The same numerical features were used to calculate the conditional forecasts based on the pre-defined demographic scenarios, treated here as given (see Section 3.3 of the current report).

4. A Delphi survey among experts

4.1. Introduction to the Delphi method

This section aims to present the Delphi method, which in a simplified form has been applied in the current study. As mentioned before, the proposed forecasting methodology for the IDEA project encompasses the elicitation of *a priori* expert knowledge on immigration processes concerning seven European countries taking part in the forecasting exercise. The expert knowledge constitutes a vital element of predictions, being eventually applied within a formal Bayesian forecasting model.

[12] The Monte Carlo standard error of the mean (for details see Spiegelhalter, 2007). It can be interpreted as the contribution of the simulation error to the uncertainty concerning the estimation of the mean (e.g. Osiewlaski, 2001).

In general, Delphi is a technique that obtains data and opinions through surveys carried out via mail, which originally stems from the applications in the US military (see eg. Dalkey, 1967). The key features of the Delphi method are (Armstrong, 1985; Rowe and Wright, 1999):

- The respondents are experts in the subject under consideration.
- The respondents are anonymous.
- Judgements are obtained iteratively: experts are asked the same questions more than once.
- Feedback for the respondents is provided: the respondents are informed about the results of the preceding round. They can formulate their opinions in order to reach a consensus. The respondents with extreme answers may be asked for the reasons for their views.
- The answers can be statistically aggregated.

The anonymity of the respondents ensures that the opinions are expressed without the social pressure of the majority or the dominant individuals in the group. The iterative procedure (two or more rounds) and the feedback concerning the general results of the previous rounds give the experts the opportunity to change their opinions in order to achieve compromise, again anonymously. The feedback comprises a simple statistical summary of the preceding round answers. It may provide additional information, such as the arguments of the respondents whose answers are extreme with respect to the average.

On the other hand, Armstrong (1985) points out that, admittedly, adding rounds brings greater accuracy and consensus with respect to the outcome, yet it is uncertain whether the gains could be greater if the number of experts was increased. Moreover, in their Delphi evaluation review, Rowe and Wright (1999) suggest that the greater number of rounds may result in a correction of the opinions to conform with the group without changing the opinion. They advise that the number of rounds should be three. This number suffices to achieve stability of the responses and reduces the risk of conformism (Rowe and Wright, 1999). The Delphi survey used in the current study comprised of two survey rounds (which is the least acceptable number), instead of the ideal three rounds, mainly due to the constrained availability of resources.

4.2. Formulation *of the questions*

In general, the Delphi technique requires preparation of a survey according to the rules that take into account insights from cognitive psychology, so as to ensure unambiguous answers. Rowe and Wright (2001: *passim*) provide evidence for a strong influence of question formulation on the answers obtained. Hence, most importantly, the questionnaires should contain a clear definition of the subject in the question. The key hints on question formulation are (*idem*):

- The question should be long enough to ensure its correct interpretation by the respondent, yet it should not be complicated and overloaded with information, but instead phrased in simple language.
- Questions should not contain emotive phrases, to avoid connotations and prejudices.
- The wording of the question, especially with respect to numbers, is also important, as it may induce the anchoring or bias effects.

- Questions should not incorporate too much or irrelevant information. With too much knowledge provided, the respondents may tend toward discarding it, and the irrelevant information may be considered relevant. Armstrong (1985: 104) suggests that "lack of information is better than worthless information."
- When the questions are formulated, it is also recommended to pre-test them with someone in order to ensure that they have the intended meaning.

In the current research problem, the questions concern predicting future immigration flows and, in particular, the structural parameters of the models employed in the analysis. A novel application of the Delphi approach in the study consists in the aim to combine the prior knowledge elicited from the experts with the quantitative data, in order to obtain forecasts. Originally, the Delphi survey alone was used as a tool for prediction-making (for a migration forecasting example, see e.g. Drbohlav, 1996).

Once the questions have been formulated, the experts for the survey can be selected. Although the evidence suggests that the expertise alone is not of a great value in forecasting (Armstrong 1985), the current task is, however, to elicit the expert judgements that will be used as a prior knowledge (expressed in the form of probability distributions) for further research, namely for combining it with the data. In general, the choice of experts should be carried out according to the following rules (for details, see Rowe and Wright, 2001):

- The experts should have the appropriate domain of knowledge.
- The combined experts' knowledge should encompass the whole problem domain, not only a particular field. Hence, heterogeneous groups of experts are preferred.
- The group should be between 5 and 20 experts. It is argued that more respondents may cause the information overload, conflicting opinions or irrelevant arguments. The number of experts should depend on the resources available and the quality of feedback expected from them, however this range is arbitrary.

One particularity of the forecasting task presented in the current study is that several questions concern subjective probabilities. That means that the experts are asked, how they perceive the future in terms of subjective beliefs or convictions about the behaviour of a particular variable, in our case, the inflow of migrants to the expert's country of expertise. Hence, the formulation of the questions requires attention with regard to proper perception of the very concept of probability.

The research on the assessment of probabilities shows that the *direct* methods sometimes can be inconsistent with *indirect* ones (see Goodwin and Wright, 1998). For instance, the estimates of odds ratios (of the form $a : b$), which are not normalised and thus may have no upper bound, tend to be more extreme than the probabilities specified within a [0, 1] interval. People also tend to view the uncertainty not expressed as subjective probabilities but rather as frequencies (Gigerenzer, 1994; Kadane and Wolfson, 1998). Moreover, people perceive problems as unique, not as the instances of a wider class of events. They pay attention to the particular and specific characteristics of the subject under consideration and forget about the context and the analogies to similar events. Gigenrenzer (1994) advises that questions about probabilities should be formulated as questions about proportions, so as to provide the wider context of the subject. This method allows also for elimination of the overconfidence of the respondent in his or her subjective probability. Only when the event under consideration is truly unique, the subjective probability should be employed directly by using the judgmental heuristics (for references, see Rowe and Wright, 2001).

The problem of overconfidence arises also when the coherence of the probabilities is considered. For two mutually exclusive and exhaustive events the probabilities should sum up to one. The general tendency is, however, that the greater the number of such exclusive and exhaustive events, the greater the chance that the sum of such 'probabilities' exceeds 100% (Armstrong, 1985). Nevertheless, the latter problem can be overcome by the means of a simple standardization of the values provided by the respondents.

Another problem that arises while assessing the judgments about probabilities and probability distributions is overconfidence of the respondents in providing too narrow uncertainty ranges. The starting question about the mean or median of the distribution may lead to the anchoring of the answer, lowering the variability and difficulties in assessment of the tails of the distribution (see e.g. Kadane and Wolfson, 1998; Rowe and Wright, 2001). The assessment of the variability may require detailed technical considerations, such as variance decomposition (e.g. O'Hagan, 1998), although in the current study a different approach is followed, described in more detail in Section 5 together with the whole questionnaire. Furthermore, in order to provide some intuition for the experts about the ideas included in the questions concerning the model parameters and their probability distribution characteristics, a visual presentation of the behaviour of the variables under consideration or the possible answers to the questions (such as the shape and direction of the trend) was proposed.

5. Elicitation of prior information

5.1. General information

In the current study, the *a priori* expert knowledge has been elicited from between six and fourteen respondents per country. The survey-based elicitation process consisted of two rounds, so as to allow for corrections and possible convergence of the initial judgments, hence, following a Delphi framework described in the previous section.

The survey concerned process characteristics (parameters of the forecasting models), rather than the processes as such (future values of migration volumes). This solution was found more straightforward, as it does not require additional re-calculations in order to transform the expert-based predictive probability distributions into the prior ones. Besides, the inference on the future values will ultimately combine data and expert knowledge, so that the predictive distributions obtained *a posteriori* would anyway differ from the ones elicited from the experts, which may lead to interpretational difficulties. In any case, the aim was to elicit expert knowledge using a natural language (or terms close to it) and visualisations of certain concept, rather than formal terms.

Unlike in the implicit assumptions made in many Bayesian literature examples (cf. Kadane and Wolfson, 1998; Dey and Liu, 2007), in the presented study expert knowledge has been elicited from migration specialists of various background, but predominantly from non-statisticians (for a thorough overview of elicitation issues in this context, see O'Hagan, 1998, and O'Hagan *et al.*, 2006)[13]. For the current study, this implied very strong limitations on the use of formal terms in such a survey (for example, 'distribution', 'variance', 'probability', 'stationarity', 'quantile', etc.). However, with respect to migration research, the area seems uncharted. As it has been noted by A. O'Hagan (1998: 22), "[…] to elicit a genuine prior

[13] Another example of a purely 'non-statistical' elicitation can be found in the study of Szreder and Osiewalski (1992), who analysed the instances of supply shortages in the then-socialist economy of Poland.

distribution […] is a complex business demanding a substantial effort on the part of both the statistician and the person whose prior beliefs are to be elicited. A Bayesian who wishes to take this task seriously finds little guidance in published work that is directly relevant to the task that he or she faces."

The questionnaires for each of the IDEA countries were prepared using the same layout, which allowed for handling the country-specific information, such as definitions of an immigrant or data collection practices. Besides, two versions of the questionnaires were prepared: English only, and a bilingual one, including, apart from the English text, a translation of it into the national language. Due to the fact that the questions concern the distributions of possible model parameters, our aim was to provide the respondents with some intuition about the possible answers using the visual presentations. In general, the questions concerned the general tendency of immigration to a particular country, the shape of the process, its volatility, the possible impact of some economic and demographic variables on immigration, as well as main directions of migration inflows. The questionnaire is described in more detail below.

5.2. Construction of the questionnaire

The questionnaire consisted of fifteen questions:

- The first question concerned the long-term (until 2025) general tendency (direction) of the future immigration flows. The figures indicating the shape of the trend (constant, as well as increasing or decreasing linear, logarithmic and logistic), along which the flows would follow, were presented. The experts were asked to choose one from the figures or to describe other type of trend.
- The second question aimed at the elicitation of the stationarity[14] characteristics of the immigration process, treated as a stochastic one. Three figures presented the example immigration processes that indicated the stationary (white noise), non-stationary (random walk) and explosive characteristics. The experts were asked to provide the chances (in terms of percentages) of occurrence of a given process or to describe their own characteristic and assign a subjective probability to it.
- The third question concerned the volatility characteristics of the future immigration process, or, more technically, how the variance of the immigration process would behave. Two example figures presented the idea of constant (stable) and stochastic (changing over time) volatility. As in the question 2 the experts were asked to provide their estimates of the probabilities of occurrence with the possibility to describe their own characteristic.
- In the fourth question the experts were asked for the estimates of the future deviations of the immigration processes from the assumed (for a given country) average immigration levels, in terms of percentage points, to be chosen from a range 10 – 1,000% or to provide their own. This question concerned the standard deviation level of the process.
- The aim of question 5 was to bring the estimate of the volatility of the variance level provided in the antecedent question. This volatility was required as a characteristic of

[14] A stochastic process $\{y_t, t=1,2,3,\ldots\}$ is called (covariance-) stationary in a weak sense if and only if its expected value and variance exist, are finite and are independent of time, and the covariance of y_t and y_s is a finite function of the term $|t - s|$ and not of t nor s alone (Greene 2003: 612). In lay terms, one can think of stationarity as 'stability' of the mean and variance of the process, which is the reason of its similar behaviour in different periods of time.

the prior distribution for the variance (see detailed description in Section 5.3). This was achieved by asking the experts for their certainty concerning the answer given in question 4. The certainty was measured on a 11-degree scale ranging from 0 (very uncertain) to 10 (almost sure).

- Questions 6 to 9 concerned the additional economic and demographic variables possibly influencing the future immigration processes (see Section 3.3). The questions aimed at providing the estimate of the character of the impact of these variables on immigration. Specifically, in the subsequent questions the experts were asked, whether faster economic growth (‘stimulant’), decreasing unemployment rate, decline in natural population growth and the decrease in the share of the productive age group (‘inhibitors’) would be associated with proportional, even faster or even slower immigration growth, with immigration decline, or whether the particular variable is not relevant for immigration process (one option to be chosen).
- In question 10 the experts were asked to provide up to three variables (in the order of importance) in their opinion potentially influencing the future immigration processes other than the ones listed in questions 6–9.
- Question 11 aimed at providing the direction of the impact of the variables described in question 10. The experts were asked to indicate whether the certain behaviour of the variable would cause the immigration to increase or decline (expressed in terms of positive and negative impact).
- Question 12 concerned three most important future source countries (directions) of inflow of the immigrants (or citizenships in the case of Hungary and France). The experts were asked to provide up to three, in their opinion, most important source countries with an indication (if relevant), whether these would likely be returning migrants.
- In question 13 the experts were asked to evaluate the expected future tendency of the directions of inflows listed in the previous question by choosing one from the options: increasing, decreasing, stable, or to describe more complex pattern.
- Question 14 aimed at providing the professional background of the experts (information not relevant for and not used in the forecasting exercise).
- Question 15 was an open-ended one, where the experts could provide the comments (concerning the merit, as well as the questionnaire itself), additional explanations or justifications for their answers with the possibility to indicate, whether their comments could be shared with the other experts in the following round.
- In addition to the above-mentioned questions, the second-round questionnaire contained summaries of first-round answers in the form of histograms (question 1, as well as 4 through 9), probabilities expressed as percentages (questions 2 and 3), or tables (questions 10 through 13). Besides, two new questions were added in the second round:
- Question N1 aimed at the assessment of the characteristics of the logistic trend (if an expert indicated so in the preceding first question), namely the likely upper asymptotic value (an upper bound) of the future immigration level and the inflection point of the trend curve.
- Question N2 concerned the impact of the most common variable listed in the first-round answers on the future immigration flow (in most cases it was the immigration policy). The question was formulated in the same manner as questions 6–9. In the second round the questions 10 and 11 were omitted as they served as a basis for the question N2 (summaries of the answers from the first round were also provided).
- The experts’ answers to the questions were summarised and then used to formulate the prior probability distributions of the model described in Section 3. The translation of

the answers to the quantitative characteristics of the densities are presented in the next subsection.

5.3. Translation of the answers into probability distributions

Herein we describe, how the knowledge elicited from the experts was transformed into the probability densities. Note that all the prior distributions of the model parameters are presented in detail in Appendix B, while the models used for forecasting are described in Section 3.

Firstly, the constants, c, were included in every model to handle the mean value of the (log-transformed) immigration levels. The priors for every country but Poland (explained in detail below) were normal with mean 0. The precision for constants in the autoregressive models were diffuse, however the information concerning the immigration policy was used. The tighter policy the majority of experts indicated in their answers to question N2 (options 'slower' or 'proportional growth'), the less diffuse prior (greater precision) was set for the constant. In the case of pro-immigration policy, hardly-informative priors were set (precision was smaller)[15]. The priors for constants in the random walk models were, in almost all cases, concentrated in 0 due to the undesirable characteristics of the process (technically, an infinite RW process with drift has an infinite expected value), and the resulting absurdity of the produced forecasts (exploding immigration flows). On one hand, this can be viewed as a drawback of the analysis, but on the other hand, as confirmed by the results (see Section 6), the characteristics of the random walk process allow to capture the specific variability in the immigration data very well.

The deterministic trend indicated by the vast majority of experts in all countries but Poland was logarithmic. The trend was included in the AR models only due to the general characteristics of the non-stationary RW process, mentioned in the previous paragraph. As almost all experts in all countries pointed out the increasing tendencies of the immigration flows, the priors set for the parameters γ in the AR models are normal with mean 0.5 and variance 1. This hyper-parameters ensure a distribution with about 30% of the probability mass below 0. The exception was the Czech Republic, where the prior set for γ was diffuse, in order to ensure reasonable results.

In the case of Poland, the logistic trend suggested by the experts was described in detail in Section 3.4. The logarithm of α in equation (13) served then as a constant, with the prior (assumed normal) elicited from the answers to the question N1: the upper bound for immigration was set to about 90,000, thus the mean of the prior was ln(90,000), and the precision defined according to the precision estimated from the experts' answers sample. The value of t^* in the equation (14) was elicited from question N1 as year 2019 (a year in which the increase would begin to slow down). The prior for the coefficient γ was assumed to follow a Beta distribution with parameters 20 and 2, and was informative, mostly due to the computational and convergence issues (Figure B6.3, Appendix B, presents the prior and posterior distributions for γ).

[15] We are perfectly aware that the adopted solution is to a large degree arbitrary. As it was difficult to formulate the policy-related question and operationalise the answers, we decided to propose a simple solution that we found sensible (tighter policy – less room for change). However, except for several cases of the random walk models, these distributions appeared not to matter so much – the priors were relatively diffuse and the data changed them anyway.

In the case of the parameters ϕ of the AR models, the normal priors were set according to the information from the answers to the question 2, parts A (stationary process, with $|\phi|< 1$) and C (explosive process, $\phi >1$). The random walk cases ($\phi=1$) in part B was treated separately, while the processes with $\phi \leq -1$ had negligible probability mass attached to them, and could be therefore ignored without a loss of generality. The answers from A and C were normalised to represent the probability mass below and above 1 respectively and then the values of the mean and precision were found using grid-searching algorithm.

The priors for the precision parameter of the model, $\tau=\sigma^{-2}$ were assumed to follow the Gamma distributions, $\Gamma(r, \mu)$, examples of which have been provided in Section 3.1. The shape parameter, r, was set to 2, which was underlain by the answers given by the experts to question 5: the average degree of experts' certainty concerning the estimates of the mean standard deviation oscillated in every country around 4–6 (medium uncertainty), hence it was justifiable to use $r = 2$ for each of them and then to control for the expected value of the precision using the scale parameter μ. Had the answers been different, than either $r = 1$, or $r = 3$ would be used, respectively depicting very high uncertainty (with answers to question 5 falling on average into the range 0–3), or, adversely, very high certainty (answers from the range 7–10).

The expected value of the precision was derived from question 4. The weighted mean of the answers, that aimed at obtaining the estimate of the standard deviation, was multiplied by 1.25 in order to eliminate the bias resulting from the usual confusion of the average absolute and standard deviations (for details see e.g. Goldstein and Taleb, 2007). The interpretation of this estimate, denote it as a, was the ratio of the standard deviation to the expected value of the underlying immigration process, according to the formula $\sqrt{Var(m_t)} = a \cdot E(m_t)$. Then, assuming the log-normal distribution of the underlying immigration process, m_t, the expected value for precision was calculated using the formula: $E(\tau) = -\log\left(1 + \frac{Var(m_t)}{(E(m_t))^2}\right)$. Finally, the value of scale parameter μ of the Gamma distribution was calculated from the equation: $E(\tau) = r / \mu$. Needless to say, the proposed procedure is merely one of the available options for operationalising the expert judgement with respect to precision. In a further sensitivity analysis, one could alternatively use Gamma distributions with other parameters, base the results only on the outcome of question 4 (which would then indicate both mean and standard deviation of τ), after examining the correlation between the answers to questions 4 and 5[16].

The values of the hyper-parameters for priors of the SV model-specific parameters, namely K, ψ and ρ (for details see the tables B(*).3 in Appendix B[17])were set in order to assure the convergence of the algorithms, however, the information delivered by them was rather vague. The prior for dummy incorporated in the models for Czech Republic was specified according to the change of the definition – a Normal prior was concentrated in –1 as it corresponded with a more rigorous definition of an immigrant until 2000. In the case of Portugal the prior set for a dummy was diffuse, in order to 'let the data speak for themselves'.

In the case of prior probabilities set on the model space (see Section 3), they were elicited from the answers to question 2 and 3, assuming the independence of answers to these two

[16] We are very grateful to Anna Żylicz for this suggestion.
[17] The asterisk (*) indicates in this context a country index (sorted alphabetically from 1 for Austria, to 7 for Portugal).

questions. First, the marginal probabilities for AR models, $p(M_1,M_3)$, and RW models, $p(M_2,M_4)$, were calculated from question 2 as summed averaged answers to options A and C, and averaged answers to B (taking into account answers from the open-ended option D), respectively. The probabilities for CV – $p(M_1,M_2)$, and SV – $p(M_3,M_4)$, models were derived from the averaged answers to points B and A in question 3 (again including the information from open-ended option C). Finally, the sought probabilities were calculated as presented in Table 2 (assuming the independency of the AR – RW and CV – SV models).

Table 2. Prior probabilities on the model space

Model type		AR	RW
	Probability	$p(M_1,M_3)$	$p(M_2,M_4)$
CV	$p(M_1,M_2)$	$p(M_1) = p(M_1,M_2) \cdot p(M_1,M_3)$	$p(M_2) = p(M_1,M_2) \cdot p(M_2,M_4)$
SV	$p(M_3,M_4)$	$p(M_3) = p(M_3,M_4) \cdot p(M_1,M_3)$	$p(M_4) = p(M_3,M_4) \cdot p(M_2,M_4)$

Source: own elaboration.

As far as the model of shares is concerned, the source directions of immigrants were chosen on the basis of the answers to question 12, after assigning ranks to the countries listed by the experts. The normal priors for constants were rather diffuse, with mean 0 and an arbitrary precision of 0.1 (in the case of Poland it was 1, in order to avoid high oscillation in the forecasts).

The priors set for the parameters of the matrix ϕ were normal with mean 0 and a rather vague precision, however, in some cases (e.g. Poland, Italy) it was required to concentrate the priors around zero due to the explosiveness of the process (and nonsense predictions).

Models for shares included trends in the cases in which the experts indicated that the shares from certain directions would behave in a specific way, e.g. would increase. The characteristics of these trends are described in Section 3.4 and were elicited from the answers to the question 13. The hyper-parameters for the normal priors were usually set so as to ensure the tendency indicated by experts, namely, with mean 0.5 and precision 1, 30% of the probability mass remaining below 0. The exceptions were Czech Republic, where the mean was moved to 1 (in order to allow for the future increase of the shares) and in the case of Hungary, where the mean for the shares from Romania was set to 0.1, again, in order to ensure the coherent behaviour of three directions' shares in the light of the experts' justifications. In the model of shares for Portugal, the prior for a dummy was again diffuse, centred at zero, in order to 'let the data speak for themselves'.

The Wishart priors for precision in the shares models assumed *a priori* an instantaneous independence of the shares from each other (indicated by off-diagonal zeros in the matrix **T**). The choice of the hyper-parameters for precision (diagonal of **T**) had an impact, however, on the posterior precision of the forecasts. Their values varied between the countries, so as to ensure the proper behaviour of forecasts, and in particular avoiding high oscillations of the predicted shares.

As far as the vector autoregressive (VAR) models with economic and demographic determinants are concerned (for details see Section 3.3.3), the prior distributions for the constants were assumed normal and diffuse. In the case of matrix of structural parameters **A**, the priors were normal. For the first parameter, α_{11}, it was obtained using the same rule as in the case of the AR models for totals, hence the normal distribution with hyper-parameters set according to the information from the answers to the question 2, parts A (stationary process, with $|\phi|< 1$) and C (explosive process, $\phi >1$), after normalisation. As far as the prior hyper-parameters for expected values of $\alpha_{12}$and α_{13} are concerned, these were obtained as follows. Firstly, every answer from A to E to Questions 6–9 was assigned a prior mean given in Table 3.

Table 3. Prior means for structural parameters from answers to Questions 6–9

Answer	A	B	C	D	E
Prior mean	1.5	1.0	0.5	–1.0	0

Source: own elaboration.

Secondly, the hyper-mean was a weighted mean of the answers given by the experts to particular questions, with positive sign in the case of the 'stimulant' (GDP growth) and negative sign in the case of the 'inhibitors' (the other variables) of the immigration processes. Then, the hyper-variance was set arbitrarily to 4.0, which was rather concentrated, however still allowed for the data to 'speak for themselves.' The other parameters were centred in 0. The diagonal ($\alpha_{22}$and α_{33}) parameters had diffuse priors (precision was set to 0.01), while for the off-diagonal elements precision was set arbitrarily to 1 (still the results were rather insensitive to this parameterisation).

The prior distribution for the precision matrix $\mathbf{T}=\tilde{\mathbf{\Sigma}}^{-1}$ was assumed to follow Wishart distribution. The hyper-parameters were matrix **P** and k=3 degrees of freedom. Using the facts that $E(\mathbf{T})=3\mathbf{P}^{-1}$ and that **P** can be decomposed into $\mathbf{P}/k=\mathbf{DRD}$, **D** – a diagonal matrix with elements interpreted as prior standard deviations and **R** – a matrix interpreted as prior Pearson's correlation coefficients (for details see e.g. Bijak, 2008b: 117), matrix **P** was constructed as follows. The first element of **D** was set so as to reflect the standard deviation resulting from the experts' answers to Question 4, and the remaining elements were set to 1. In the case of matrix **R**, every answer from A to E to Questions 6–9 was assigned a prior mean (of the corresponding correlation coefficient) given in Table 4.

Table 4. Prior means for precision parameters from answers to Questions 6–9

Answer	A	B	C	D	E
Prior mean	0.5	1.0	0.5	–0.5	0

Source: own elaboration.

Then the weighted means with signs set according to the role of the variable ('stimulant' or 'inhibitor') for every variable represented the prior correlation. Needless to say, the diagonal parameters of **R** were ones.

Tables 5, 6 and 7 present the summary of the prior distributions used for models of totals, shares and demographic and economic variables impact, respectively. They sum up briefly, whether the applied distribution was informative and whether it was based on the expert knowledge. The exact hyper-parameters assumed and calculated for every country are included in country-specific Tables B(*).6 in Appendix B.

Table 5. Prior distributions for model of total immigration flows – a summary

Parameter	Distribution	Informative	Expert knowledge
c	Normal	Yes/No*	Yes/No*
ϕ	Normal	Yes	Yes
γ	Normal (Beta)**	Yes	Yes (No)**
$\tau=\sigma^{-2}$	Gamma	Yes	Yes
K	Normal	Yes	No
ψ	Normal	Yes	No
ρ^{-2}	Gamma	Yes	No
dummy	Normal	Yes/No	No

* Depending on the model type and/or stability of forecasts.
** Prior distribution for the logistic trend coefficient.

Source: own elaboration.

Table 6. Prior distributions for model of shares of the immigration flows – a summary

Parameter	Distribution	Informative	Expert knowledge
c	Normal	No	No
ϕ	Normal	Yes/No*	No
T	Wishart	Yes	No
b	Normal	Yes	Yes

* Concentrated distributions in such cases, where the forecasts were unstable.

Source: own elaboration.

Table 7. Prior distributions for models of impact of the economic and demographic variables – a summary

Parameter	Distribution	Informative	Expert knowledge
c	Normal	No	No
A	Normal	Yes/No*	Yes/No*
T	Wishart	Yes/No*	Yes/No*

* Informative and based on expert knowledge in the case of the immigration equations.

Source: own elaboration

6. Results of forecasts: An overview

6.1. Modification of prior beliefs in the light of data

With respect to the interpretation of the results of Bayesian forecasts, a key issue is, how and to what extent did the quantitative data modify the prior knowledge obtained from the experts using a Delphi elicitation described in the previous sections. In this context, two problems are crucial, related to the beliefs, firstly, in the probabilistic model M_i driving the processes under study, and secondly, in the model parameters as such (θ). Both these issues are illustrated in more detail in country-specific Tables B(*).5 and Figures B(*).3 in Appendix B.

The most important observation with respect to the model selection out of the possible space $M_1 - M_4$, defined in (1a) – (1d) in Section 3.3, is that in a majority of cases the data gave clear preference to a simple constant-variance random walk model with drift (M_2). For Austria, the Czech Republic, France and Italy the results were unambiguous, with the *a posteriori* probability of its selection given the data, $p(M_2|x)$, exceeding 0.95. For Hungary and Portugal, the autoregressive models with trend (M_1) also appeared to play some role, with an almost 50-50 split between the two models in the former case, and with $p(M_2|x)$ = 0.63 and $p(M_1|x)$ = 0.37 in the latter one, $p(M_1|x)$ being here slightly less than the respective prior probability $p(M_1)$ = 0.38. For Poland, in turn, two random walk models have won the competition, namely the one with the stochastic variance (M_4) and with constant variance (M_2), and the respective posterior model probabilities were equal 0.53 and 0.47. Clearly, all these results are specific to the space of the models under consideration, but at the same time appear relatively robust to the prior probabilities elicited from the experts.

For migration researchers, a conclusion that population flows do not have a 'well-behaved', which means elegantly stable (or stationary), character, is not a novelty (for more discussion, see e.g. Pijpers, 2008). However, modelling and forecasting non-stationary variables as if they were stationary ('orderly'), was a common practice in migration prediction-making (*idem*). This is as a serious methodological drawback, likely leading to very high forecast errors. A recent, spectacular example of the latter is the forecast of immigration to the United Kingdom after the 2004 enlargement of the European Union, prepared for the British Home Office (Dustmann *et al.*, 2003), which explicitly assumed stationarity of the forecasting model (*idem*: 28, 68). The inflow of immigrants from the ten new EU member states has been envisaged at as "between 5,000 and 13,000 immigrants per year up to 2010" (*idem*: 58), which underestimated true inflows by well over than an order of magnitude [18].

With respect to selected parameters of the forecasting models, in many cases the prior distributions have been visibly modified by the data, which illustrates the Bayesian mechanism of 'synergy' of *a priori* judgements and observations. For example, the precision parameters generally (with the exception of Portugal) acquired higher values *a posteriori* than *a priori*. This suggests that including the expert knowledge allows for higher uncertainty assessments than data alone. Similar conclusion was reached by Bijak (2008b), who advocated that lower predictive precision of migration might be in many cases more realistic than the higher one, yielded by the models exclusively based on the trend extrapolation and devoid of expert knowledge. As to the autoregressive terms ϕ in trend models, a tendency was observed that the posterior distributions of ϕ were more concentrated than the priors, and mostly shifted either towards the unity (especially for Austria, but also to some extent for Hungary, Italy, Portugal), or towards minus one (Poland), with some non-negligible probability of non-stationarity, $p(|\phi| \geq 1|x)$. In the Polish case the process was thus additionally equipped with traces of 'oscillating' features.

[18] For example, the Polish and British Labour Force Surveys estimated respectively 140,000 and 198,000 immigrants to the UK coming in the year 2006 alone, and only from Poland (Grabowska-Lusińska and Okólski, 2008: 32).

For the trend coefficients γ, no radical changes between the prior and posterior distributions were observed, with the exception of Poland, for which, however, γ played a different role due to the presence of the logistic trend (see Sections 3.4 and 5.3). In the Polish case, the data shifted the distribution of γ from concentrated below one to above zero, which was the result of applying strong Beta prior for this parameter, in order to ensure a smooth passage from the lower to the upper asymptote of the logistic function. Finally, the random walk constants c (drift terms) have been either identified by the data themselves from hardly informative prior distributions, or, in exceptional cases (the Czech Republic and Italy) the data slightly modified upwards these priors, which were very much concentrated around 0, following $N(0, 0.05^2)$. Under this assumption, it is highly unlikely *a priori* (with probability smaller than 0.05) that the **average** annual migration change would exceed ±10%. It has to be noted that both exceptions concerned countries with exploding migration trends, which had to be kept at reasonable levels by applying very strong prior assumptions, which under other circumstances could seem somewhat artificial.

6.2. Migration forecasts until 2025 for selected European countries

The forecasts prepared in the current study have been ultimately obtained either by using the random walk models M_2 having the highest probability *a posteriori* (for Austria, the Czech Republic, France and Italy), or by averaging of forecasts yielded by various models (M_2 and M_1 for Hungary and Portugal, M_2 and M_4 for Poland). Hence, the results presented in the current section, and in more detail in Figures and Tables B(*).1 in Appendix B, refer to either formally-selected or averaged forecasts, depending on circumstances.

Throughout this section, the predictions are presented in terms of central tendencies, which are medians from the respective predictive distributions. As mentioned in Section 3.5, location parameters, such as medians or quantiles, are much more robust statistics than moment-based characteristics, for example means or standard deviations, the latter being very sensitive to the presence of outlying observations. The uncertainty spans, in turn, are based on symmetric quantiles from the predictive distributions. Hence, the 50-percent ranges are based on the predictive quartiles (lower and upper), the 80-percent ranges on the quantiles of rank 0.1 and 0.9, and the 90-percent ranges on the quantiles of rank 0.05 and 0.95. However, the latter ones should be used merely for indicative purposes. As noted by Lutz *et al.* (2004: 37), "[t]he 80 percent intervals are far more robust to the technicalities in the forecasting methodology than the 95 percent intervals."

The ultimate selection of a given trajectory for further policy-relevant analysis will strongly depend on the nature of the decision problem. For example, in some cases, the underestimation of future migration inflows will have more profound consequences (be more costly) than its overestimation, as for example in assigning budgets to migrant integration programmes. In such instances, it would seem rational to use the above-median variants, in order to "stay on the safe side" (e.g., the upper quartile, or the 90% quantile, depending on the magnitude of expected losses given various degrees of overestimation). In other examples, overestimation can be more problematic, as in the case of the inflows required to fill the local labour market shortages. Then, it would be more cautious to use below-median forecast variants as input in the decision making process (e.g., the lower quartile, or the 10% quantile). This recommendation follows a tacit assumption that the decision maker is risk-averse. Nevertheless, we believe that in the public sphere some degree of cautiousness and risk aversion is a desirable feature, especially with respect to the long-term planning and policy-making.

In all cases, the *ex-ante* uncertainty assessments reflect our beliefs, based on the expert knowledge and statistical data, that the future migration inflow will fall in a given interval with a pre-defined (subjective) probability. Especially note that the probability of migration **not** being higher than the upper values of the 50-percent, 80-percent and 90-percent intervals, equals respectively 0.75, 0.90 and 0.95. In general, all these quantile-based *ex-ante* assessments of the predictive uncertainty are illustrated in said Figures B(*).1, and the 50-percent ranges for selected years are additionally listed in Tables B(*).1, in Appendix B.
Given the assumptions listed in the previous section, it is expected that for **Austria**, with respect to the central (median) tendency, the total yearly immigration volume would still increase, from 106.9 thousand in 2007, through 151.8 thousand in 2016, to reach about 215.3 thousand by the end of the forecast horizon. The related 50-percent predictive intervals cover the range between 78.4 and 293.6 thousand for 2016 and between 69.6 and 660.0 thousand for 2025. The 80- and 90-percent intervals naturally cover much larger numbers, especially towards the end of the forecast period.

The results obtained for the **Czech Republic** are characterised by the expectations of relatively higher uncertainty than the one envisaged for Austria. From the initial level of about 104.4 thousand immigrants in 2007, the median tendency indicates an increase to 135.9 thousand in 2016 and to 176.3 thousand in 2025. However, the 50-percent predictive intervals show a rapidly increasing forecast uncertainty: from between 52.0 and 348.0 thousand in 2016, to between 42.2 and 715.0 thousand in 2025. It can be concluded that the judgements of the Czech experts coupled with the past trends observed in the data suggest that especially the more distant future of immigration to the Czech Republic is really uncertain.

For **France**, the forecasted immigration inflows are characterised by an increasing tendency. From the initial level of 207.5 thousand people in 2005, the median inflows increase to about 300 thousand in 2015 and to 442 thousand by the end of the forecasting horizon. The 50-percent predictive intervals indicate an increasing uncertainty towards 2025. In 2015 this interval ranges from about 180 to 509 thousand, whereas in 2025 its bounds equal 184 and 1,056 thousand immigrants, respectively. The 80- and 90-percent intervals are much wider, giving clear indication of an increasing uncertainty towards the forecasting horizon.

The forecasts for **Hungary** depict the expectations of rather steady developments of future migration inflows, at least in terms of median trajectories and the related 50-percent predictive intervals. In particular, with respect to the central tendency, the inflows are expected to increase only very slightly: from the initial 21.5 thousand in 2006, through 22.2 thousand in 2016, to 23.2 thousand in 2025. The 50-pecernt intervals are also relatively narrow, covering the span between 12.5 and 38.2 thousand people expected to immigrate in 2016 and between 10.3 and 46.6 thousand people at the end of the forecast horizon (in 2025).

The forecasts for **Italy**, in turn, are even more uncertain than the ones presented before for the Czech Republic. This is due to two major factors: a long and steady increase of migration observed in the past, and dramatic expectations of the experts resulting in much weight put by them on the explosive nature of the process (as much as 24% of the answers, versus 37% for the stationary character and 38% for the random walks). These circumstances, coupled with the unique availability of a long data series, resulted in the median expectations on the continuation of the past increase of inflow: from 305.0 thousand in 2005, through 369.5 thousand in 2016, to 433.7 thousand in 2025. The 50-percent intervals indicate wide uncertainty spans, with their upper bounds reaching 839 thousand people in 2016 and 1.4

million in 2025. Naturally, such values as the latter one should be judged as hardly plausible. Hence, these forecasts should not be interpreted in terms of precise values but, if anything, at most in terms of the orders of magnitude. Given the prior knowledge elicited from the experts, as well as data trends, the presented results should be seen as primarily indicating the extremely high degree of uncertainty with respect to the future migration inflows to Italy.

As compared with Italy or even the Czech Republic, the forecast results for **Poland** are relatively stable. In the light of the median variant, the permanent immigration[19] to Poland is expected to increase from the initial 15.0 thousand people in 2007, through 28.3 thousand in 2016, to 53.1 thousand people in 2025. The 50-pecernt intervals are relatively narrow, albeit widening, and cover the range between 15.9 and 53.6 thousand people forecasted to immigrate in 2016 and between 20.5 and 157.9 thousand in 2025.

Similarly, the results obtained for **Portugal** seem plausible from the demographic point of view. The median trajectory indicates that immigration is expected first to slightly decrease, from 30.7 thousand in 2006 to 29.7 thousand by 2008–2009, and then to continuously increase up to 33.9 thousand in 2016 and to 40.1 thousand at the end of the forecast period (in 2025). The respective 50-percent intervals encompass the inflows between 15.9 and 74.6 thousand people in 2016, and between 13.4 and 120.6 thousand foreseen for 2025.

In general, the forecast results obtained for the IDEA countries can be summarised as follows. In all cases, migration appeared to be much more likely generated by the non-stationary processes (random walks), than by the models equipped with a trend and autoregressive features. This outcome is not surprising, given the nature of phenomena under study (cf. Bijak, 2008b), especially that migration is likely the most uncertain component of population dynamics. One of the features of the random walk model is that it cumulates all the random shocks – or uncertainty (embodied in the error term, for technical details see e.g. Greene, 2003: 593) – from the starting point of the process under study. The random walk is thus said to have ‘long memory’. This ‘summing up’ of the uncertainty results in the increase of the width of predictive intervals over time, strongly implying the length of a sensible forecast horizon, which ideally, in our opinion, should cover one decade at the most. After ten years, the bounds of predictive intervals (80-percent, and in many cases also 50-percent) become too high to offer any meaningful information for the forecast users. For this reason, the interpretation of the forecasts yielded by the current research should be limited to the ten-years’ horizon, and the remaining period should be used merely as an illustration of the increase of the uncertainty associated with migration over time[20]. This conclusion is especially valid for Italy: for the reasons mentioned before, such as the features of the past trends and the experts’ expectations, the plausible forecast horizon has to be short. Otherwise, after 2020, the upper bound of the 50-percent predictive interval exceeds one million immigrants to Italy, which can be seen as far too high to be reasonable. Notwithstanding, in all cases the one-decade-ahead forecasts are marked by frames in Figures B(*).1, Appendix B.

[19] It is an open issue, whether the questions have been intuitively interpreted by all the experts as pertaining to permanent immigration only. It might have happened that, despite clear indications in the survey, some experts cognitively ‘broadened’ the definition and gave replies which in reality would reflect some other types of flows. Nevertheless, as we have found no satisfactory solution to this problem, the awareness among migration experts of the actual meaning of various definitions in use might be worth investigating in further research focusing on Poland. Nevertheless, we are grateful to Anna Żylicz for drawing our attention to the problem of potential misinterpretation.

[20] Interestingly, the postulate that demographers should not make migration forecasts for a longer period than ten years ahead, has been made by J.Z. Holzer already half a century ago (cf. Holzer, 1959).

6.3. Conditional forecasts of the most important migration directions

As mentioned before, in almost all of the countries under study the experts indicated three significant directions of inflow, defined either in terms of origin countries, or citizenships. The only exception was Poland with two only two major directions of inflow. The results of forecasts of the identified important immigration directions for particular countries are summarised below. In all cases, the presented forecasts are conditional on the median values of the predicted shares, as well as of global migration volumes, the latter presented in Section 6.2. For detailed sources of particular numbers quoted from the data sample, see Table A2.1 in Appendix A.

For **Austria**, the most vital inflows are expected from three countries or regions: Germany, the former Yugoslavia[21] and Turkey. In the last observation period (2007), the total inflow of 106.9 thousand people was comprised in 19% of immigration from Germany (20.4 thousand), in 13% from the countries of the former Yugoslavia (13.5 thousand) and in 5% from Turkey (5.4 thousand). The median forecasts, prepared on the basis of data for 1996–2007, indicate that in the case of immigration from Germany, a slight decline in the shares is expected, to the levels of 12% after a decade (in 2016) and to 11% in 2025. At the same time, two other inflows would increase: from the former Yugoslavia to 28% by 2016 and 32% by 2025, while from Turkey – respectively to 11% and 13%. In 2016, these shares would correspond to the median forecast of inflow of 18.7 thousand people from Germany, 42.3 thousand from ex-Yugoslavia and 16.4 thousand from Turkey, out of 151.7 thousand immigrants. By the end of the forecast horizon (in 2025), the respective volumes would amount to 24.2, 68.4 and 27.9 thousand people, out of 215.3 thousand.

The most important immigration directions envisaged for the **Czech Republic** were: Slovakia, Ukraine and Vietnam. In 2007, out of the total of 104.4 thousand immigrants, 39% came from Ukraine (40.3 thousand), 14% from Slovakia (14.2 thousand) and 10% from Vietnam (12.6 thousand). The median forecasts, based on the 1993–2007 data, indicate an expected increase in the shares of immigrants from Vietnam, to 20% in 2016 and 23% in 2025. The share of people coming from Ukraine would stabilise around 36% throughout the forecast horizon, while from Slovakia – increase to 18–19% in the first decade of the forecast, and then decrease below 17% by 2025. In terms of numbers, the median forecasts for 2016 indicate 48.8 thousand people coming from Ukraine, 27.4 thousand from Vietnam and 24.4 from Slovakia (out of the total of 135.9 thousand people). By 2025, the respective volumes of annual inflows would change respectively to 64.3, 40.8 and 29.6 thousand people, corresponding to the total of 176.3 thousand.

In the case of **France**, the most important directions of inflows indicated by the experts were China, Morocco and Turkey. The data available were expressed in terms of the citizenship. In 2005, the total number of immigrants granted long-term settlement permits was 207.5. The Moroccans comprised 10% (21.5 thousand), Turks and Chinese – 4% (about 8 thousand) each. According to the median forecasts, computed on the basis of the 1994–2005 data, the share of the immigrants from China is expected to increase from 3.6% in 2006, through 10% in 2016 to about 14% in 2025. In the case of Turks, the shares are increasing from 3.9% in 2006, about 6.5% in 2016 to 7.2% in 2025. The share of Moroccans is rather stable, almost 12% in 2006 and then increasing and slightly oscillating around 14% towards 2025. These shares imply the following median forecasted inflows: in 2006, about 25.5 thousand of

[21] For the purpose of the current study, comprised of all ex-Yugoslav republics: Bosnia and Herzegovina, Croatia, the FYROM (Macedonia), Montenegro, Serbia (including Kosovo) and Slovenia.

Moroccans, 8 thousand immigrants from China and 8.4 thousand from Turkey. In 2016 these amount to 44 thousand, 31.4 thousand and 20.5 thousand, respectively. By 2025, the respective numbers would correspond to 61, 62 and 32 thousand people, whereas the total annual inflow of immigrants would equal 442 thousand.

For **Hungary**, the three important directions of inflow identified by the experts, expressed in terms of citizenships, included Romania, Ukraine and Serbia (due to the nature of the time series, the latter treated jointly with Montenegro and Kosovo). In 2006, out of the total of 21.5 thousand immigrants, 6.8 thousand (32%) were Romanians, 2.4 thousand (11%) – Ukrainians, and 1.1 thousand (5%) – citizens of Serbia and Montenegro. According to the median forecasts prepared on the basis of data for 1992–2006, the share of Romanian immigrants is expected to slightly decrease from 43% in 2007, through 39% in 2016 to 38% in 2025. For Ukrainians, an increase from 15% in 2007 through 19% in 2016 to 20% in 2025 is envisaged, while for the citizens of Serbia and Montenegro – the most dynamic increase, from 6% at the beginning, through 12% mid-way to 15% by the end of the forecast horizon. In 2016, these shares imply the median forecast of inflow of 8.7 thousand Romanians, 4.1 thousand Ukrainians and 2.7 thousand Serbs and Montenegrins (out of 22.2 thousand immigrants altogether), whereas in 2025, the respective numbers would equal 8.8, 4.6 and 3.5 thousand people, out of 23.2 thousand in total.

For **Italy**, the answers of experts indicated Romania, Ukraine and Morocco as three most important countries of origin of immigrants, although ultimately it has been decided to replace Ukraine with Albania, due to the limited availability and very unstable, explosive features of the former data. Even in the cases under study, however, the available data series are very short, covering only ten years (1995–2004). In the last observation period, the total registered inflow of 414.8 thousand people was comprised in 15% of immigration from Romania (64.3 thousand), in 9% from Albania (37.2 thousand) and in 7% from Morocco (31.0 thousand). According to the median forecasts, the latter share inflow is envisaged to remain stable at just below 8% throughout the forecast horizon, which would correspond to 28.4 thousand immigrants from Morocco in 2015 and 33.5 thousand in 2025. At the same time, the share of inflows from Romania would rapidly decline to about 10% and remain relatively constant, while the one from Albania – stabilise just around 12% starting from 2008. In absolute terms, however, these inflows would follow an increasing tendency: from Romania to 38.0 thousand people in 2015 and 45.2 thousand in 2025, and from Albania to 44.2 and 52.4 thousand people in the same years.

As mentioned before, for **Poland** two most important directions of future inflows were identified by the experts: Ukraine (immigration) and United Kingdom (return migration), both well ahead of all other countries of origin. In 2007, out of 15.0 thousand immigrants registered for permanent residence, 777 people (5%) came from Ukraine and 3.9 thousand (26%) from the UK. The median forecasts, prepared on the basis of the 1990–2007 data, indicate that the share of immigrants from Ukraine would first increase to 10% already in 2008, and to over 11% in 2013, only to decline below 10% in 2016 and ultimately to 6% in 2025. The return migration from the UK would follow a similar pattern: from below 15% in 2008, through a peak over 18% in 2014, below 17% in 2016, down to below 10% in 2025. Both these trajectories are a result of including in the forecast a trend suggested by many experts in the survey (see Section 3.4 for details). The shares for 2016 would correspond to 2.8 thousand immigrants from Ukraine and 4.8 thousand (mostly) return migrants from the United Kingdom, among 28.2 thousand immigrants altogether. For 2025, the respective numbers would equal 3.3 thousand for Ukraine and 5.1 thousand for the UK, out of the total of 53.1 thousand permanent immigrants in total.

For **Portugal**, the key inflows are expected from three Portuguese-speaking countries: Brazil, Cape Verde and Guinea-Bissau. In the last observation period (2006), the re-calculated total inflow of 30.7 thousand people in 18% comprised immigrants from Brazil (5.4 thousand people), in 5% from Cape Verde (1.4 thousand), and in 2% from Guinea-Bissau (662 people). The median forecasts yielded by the models estimated on the basis of the (again, re-calculated) 1992–2006 data indicate in all cases a slight increase until about 2013–2016 and a stabilisation thereafter, at the levels of ca. 13% for Brazil, 8% for Cape Verde and 3.5% for Guinea Bissau. Translated into numbers, these shares would correspond to the median forecast of 4.3 thousand immigrants from Brazil in 2016, increasing to 5.1 thousand in 2025, then of 2.7 thousand people coming from Cape Verde, increasing to 3.2 thousand, and 1.2 thousand people from Guinea Bissau, with an only slight increase to 1.4 thousand by the end of the forecast horizon. The respective totals predicted for the two mentioned years would amount to 33.9 thousand (2016) and 40.1 thousand people (2025).

The detailed results are presented in Figures and Tables B(*).2 in Appendix B. As mentioned in Section 6.2, all forecasts made beyond the horizon of one decade should be treated only indicatively, due to the nature of the processes under study and the related uncertainty spans. Moreover, it is worth bearing in mind that all the presented results refer to the median forecasts of both global inflows and shares. Naturally, the forecasted shares can be alternatively applied to other predicted trajectories of immigration inflows. However, the predictive uncertainty of shares appeared to be so high, that the analysis of different combinations of quantile-based predictions became meaningless. Besides, as mentioned in Section 3.5, the transformations of the forecasted variables do not allow for a coherent interpretation of the results in terms of predictive intervals for shares. Therefore, all the forecasts of volumes of different migration inflows offered in this section have to be seen as **conditional** on the median predicted trajectories of shares, and, as such, depict in fact scenarios rather than forecasts.

6.4. Impact of demographic and economic variables

With respect to the analysis of the impact of additional economic and demographic variables, the outcomes of Lindley's tests described in Section 3.3.3 show a rather clear pattern. Firstly, the two economic covariates considered (GDP growth rate and unemployment rate in the receiving country) appeared to have hardly any influence on migration, even when the data were supported with the expert judgement. The only positive exceptions concern the economic growth in France (instantaneous) and Portugal (with one-year time lag), however, in both cases, the hypothesis of the lack of impact could be rejected only at the probability level of 0.1. A tentative conclusion might be that available data in general do not allow for a formal inference on the interrelations between the most important economic determinants (cf. Jennissen, 2004) and immigration.

Moreover, the impact of the two demographic variables (the natural population growth rate and the share of the productive-age population group in the destination country) was found significant in most of the cases, except for Portugal. For Austria, significance (at the 0.1 level of probability) concerned the lagged share of the productive-age population only, while for the five remaining countries various combinations of lagged and/or instantaneous influence of both demographic covariates was found at various probability levels (including 0.05, especially for Italy, and even 0.025, in some rare cases like France). Again, a tentative conclusion might indicate an important role of the two demographic factors in shaping immigration flows, at least to the extent, which would be possible to detect using the limited data at hand and expert judgement. The detailed results of tests are offered in country-specific Tables B(*).7 in Appendix B.

Following the results of the tests, conditional forecasts of immigration were calculated on the basis of the deterministic scenarios for the demographic covariates assumed from the recent Eurostat's projection (see Section 3.3.3 for details). The conditional forecasts were calculated for six countries, thus with the exception of Portugal, where no significant impact of demographic variables was found. In all cases the forecasting models were obtained following the 'from general to specific' principle, based on the test results. Thus, for Austria, a reduced, two-dimensional VAR model was used, comprising only log-transformed immigration and the share of the productive-age population. For the remaining five countries, the initial, three-dimensional models were applied, as the tests did not indicate that any of the variables could be safely removed from the general modelling framework. For all cases except Portugal, the conditional forecasts can be viewed in the relevant Tables B(*).8 and Figures B(*).6 in Appendix B.

Despite the explanatory potential of the demographic migration determinants under study, the conditional forecasts yielded on their basis, however, remain far from being satisfactory from the decision-making point of view. In the two cases with the "best" test results (France and Italy), the forecasts very quickly reach implausible values with very high uncertainty spans. Hence, it seems that even if there is significant impact of the demographic covariates, the price to pay can be extremely high predictive uncertainty, confirming the earlier suggestions of Bijak (2008b).

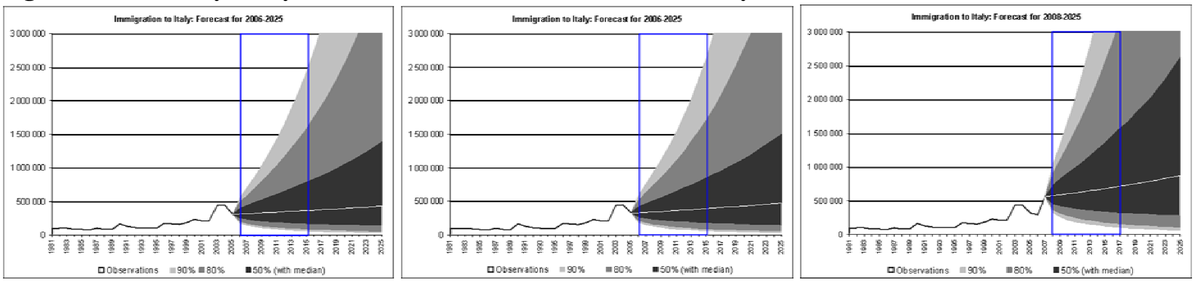
Moreover, even despite of significance indicated by the test results, the *a posteriori* estimates of model parameters in some cases can either have counter-intuitive signs (Austria), or their combination can lead to counter-intuitive (in this case, decreasing) conditional forecasts (Hungary and Poland). This means that the positive outcomes of significance tests alone do not guarantee reliable scenario-based predictions, and that the latter need to be treated with utmost caution.

6.5. Sensitivity analysis: selected aspects

Bayesian perspective adopted for the forecasts preparation requires the sensitivity analysis of the results to the assumptions made. The complexity of the problem at hand, however, precludes analysis of every aspect of the exercise. Hence, the results of the sensitivity testing in three dimensions are presented: the sample case of sensitivity with respect to the data at hand (Italy), the incorporation of the time trend to the AR model (the case of France), and finally the sensitivity with respect to the operationalisation of the experts judgments concerning the variability of the future immigration processes (questions 4 and 5).

In the case of Italy there were three data sets available: (1) final data for 1981–2005, (2) provisional data for 1981–2005, where two last observations were provisional with possible over-estimation of the figures, and (3) data for 1981–2007, where four last observations were provisional and were possibly affected by the regularisation process (for details see e.g. Cangiano, 2008). Figure 2 presents forecasts based on these three data sets from the RW-CV model.

Figure 2. Sensitivity analysis to the data set used – the case of Italy



Source: own elaboration.

We observe a slight increase in the median and inter-quartile ranges when comparing the forecasts from the 1st and 2nd data set and a large increase in uncertainty (together with the shift in median) in the case of the 3rd data set in comparison to the first two. In terms of the model parameters, the difference between the first two and the third was visible in the τ parameter for precision – the posterior distribution was slightly more concentrated and shifted towards 0, hence the expected precision was smaller, ultimately resulting in greater uncertainty of the forecasts.

In the case of France, the sensitivity of the model fit to the data (and thus the choice of the model by the Carlin-Chib procedure) to the inclusion of the trend in the AR model was investigated. Two sets of the $M_1 - M_4$ models were estimated, one with the AR-CV and AR-SV models with linear trend incorporated and another set without the trend. Then the Carlin-Chib procedure was applied in both sets separately in order to find the best model, the same prior probabilities on the model space were assumed. In both cases the best fit to the data in terms of the posterior probabilities was the RW-CV (the probability for this model in both cases was around 0.99). Moreover, the final results were insensitive to the starting model choice (i.e. the model, in which the algorithm starts to generate samples) of the procedure.

The third part of the sensitivity analysis carried out was with respect to the operationalisation of the expert knowledge in the construction of the prior densities for the precision parameter τ. The assumptions concerning the precision were crucial for the assessment of the uncertainty of the forecasts, thus the analysis was done for each country. Three different hyper-parameterisations were employed: A – the original parameterisation (as described in Section 5.3), B – where the hyper-parameters of the Gamma distribution were calculated from the two equations, one for the expected value of the precision equal to that of the experts, and second for the relative standard error of the precision equal the relative standard error of the experts answers given to the Question 4, and C – the diffuse prior, i.e. Gamma distribution with parameters $\Gamma(0.01, 0.01)$, implying vague precision. The results of the analysis of the precision parameter and forecasts are presented respectively in Figures B(*).4 and B(*).5 in Appendix B.

As far as the precision parameters are concerned, the posterior densities resulting from the A – C priors were very sensitive (in a sense that the shape and location parameters differed) in the case of Austria, France and Poland, the shape but not the location parameter differed in the case of the Czech Republic and Hungary and the location parameter but not the shape differed in Italy (note, that in this case the posteriors in A and B did not differ significantly from each other). In the case of Portugal, the posteriors were almost insensitive to the assumed priors.

The sensitivity of the forecasts in terms of the differences in the median of the predictive posterior distribution was observed in the case of the Czech Republic, France and Italy. Nevertheless, the most important was the sensitivity of the forecasts uncertainty. The largest uncertainty was observed in the case of Austria, where the inter-quartile range in 2017 of the B parameterisation was twice as big as for A and three times larger than for C. Another sensitivity to the prior assumptions was detected in the case of Poland, where the 3rd quartile in 2017 ranged from about 75,000 in C, through 90,000 in A, to more than 100,000 in B parameterisation. In most of the other countries (but the Czech Republic), although the difference between the informative posteriors (A and B) and diffuse one (C) were detected, there was no significant sensitivity of the results to the expert knowledge parameterisation (i.e. between A and B). In the case of the Czech Republic, the results were insensitive to any of the assumed prior structures.

In all cases but Portugal, the expert knowledge turned out to contribute to the increase of the uncertainty of the forecasts. Table 8 summarises the results of the analysis.

Table 8. Prior distributions for models of impact of the economic and demographic variables – a summary

	AT	CZ	FR	HU	IT	PL	PT
Sensitivity of the precision parameter							
Sensitivity of the **shape** / **location** parameter of the Gamma posterior distribution for □	Yes/Yes	Yes/No	Yes/Yes	Yes/No	No/Yes	Yes/Yes	No/No
Sensitivity of the forecasts							
Sensitivity of the median	No	Yes	Yes	No	Yes	No	No
(A & B) vs. C sensitivity	Yes	No	Yes	Yes	Yes	Yes	Yes
A vs. B sensitivity	Yes	No	No	No	No	Yes	Yes
Does expert knowledge **I**ncreas or **D**ecrease uncertainty?	Increase	Increase	Increase	Increase	Increase	Increase	Decrease

Source: own elaboration.

7. Concluding remarks and lessons learnt

In the methodological aspect, the forecasting exercise presented in the current study was aimed at moving towards greater synergy and coherence in migration predictions, which we believed could be obtained by combining qualitative and quantitative information within a formal framework of Bayesian probabilistic models. In order to achieve this, two methods were applied jointly: a Delphi survey, intended to yield expert-based information *a priori*, as well as formal econometric and time-series modelling within a subjectivist Bayesian setting. The methodological novelty of the proposed framework consists thus in taking advantage of the combined features of both approaches in such a difficult field of application as international migration forecasting.

The outcomes of this exercise can be summarised in the following points:
1. Migration is hardly predictable. Strongly in line with the prior intuition of migration researchers, the processes of population inflows under study appeared to have barely

predictable nature, as indicated by an almost universal data preference for the non-stationary random walk models over the 'well-behaved', stationary models with trend[22]. As suggested before, modelling and forecasting non-stationary variables as if they were stationary is a serious methodological flaw and can lead to very serious forecast errors, as well as to misguiding forecast users.

2. Uncertainty matters. Especially given the above conclusion on the nature of migration, an attempt of its precise prediction in numerical terms is doomed to fail. Nevertheless, following what is currently becoming state-of-the-art in demographic forecasting, the predictive uncertainty can be embraced by using the stochastic approach (Keilman, 1990), which presents and quantifies the randomness in an explicit manner. Presenting deterministic forecasts instead can be seen as merely hiding the problem of predictive uncertainty, which, however, does not make the problem disappear. It has to be noted that the methodology used (deterministic versus stochastic) does not alter the nature of the process, but only the way the expectations for the future are formed and presented (as single scenarios, or as sets of predictive distributions). Hence, preparation and presentation of migration predictions in a deterministic fashion gives the decision makers a false sense of certainty, which they should definitely avoid in order to make informed decisions.

3. Expert knowledge matters – but not everywhere. In the current study, the impact of judgmental information elicited from country-specific experts in the Delphi survey, aimed at supplementing weak sample-based information from short data series, was varied. On one hand, expert knowledge (*a priori*) appeared to be very important in the estimation of the model parameters, especially with respect to the expected precision (or variability) of forecasts, or in such heavily-parameterised models, as the vector autoregressive (VAR) ones. On the other hand, the impact of subjective expertise was much less profound with respect to the model selection, that is, to the determination of the nature of the processes under study, among the possible alternatives. The dominant selection of random walk models, often having quite low probabilities *a priori*, was to a large extent independent from the expectations of experts. It seems to indicate that the uncertain and hardly predictable character of migration flows may be their immanent, more general feature rather than just a characteristic of a particular forecasting model. The latter conclusion also coincides with the results of earlier studies (Bijak, 2008b).

4. Forecasts with too long horizons are useless. Confirming some earlier suggestions (e.g., Holzer, 1959; Bijak, 2008b), the sensible horizon of migration forecasts should be limited to five to ten years at the most. This is due to the nature of the processes under study – if they are indeed non-stationary (which may be often the case, as in the random walk examples), then their uncertainty is growing over time. After this period the predictive intervals become too large to offer any meaningful information to the decision makers. A resulting research problem, although remaining beyond the scope of the current study, is, what to do with migration assumptions in population predictions prepared with a longer horizon, for example of half a century.

[22] Predictability is here understood as a degree, to which the future uncertainty about a given phenomenon can be assessed using probabilistic models. If the phenomenon was 'well-behaved' (stationary), it would be roughly speaking well-predictable. On the contrary, the non-stationary processes can be predicted only in rough terms (as in random-walk-type processes), or hardly at all (explosive features). The results of the current study suggest that migration at best belongs to the second category.

5. Impact of migration covariates may be difficult to detect. Due to the shortness of time series and hardly predictable nature of migration, significant impact of especially economic variables was found very limited, if any. This is in line with the results of some of the earlier studies (Bijak, 2008b). On the other hand, even despite the significant impact of the demographic covariates, in many cases the counter-intuitive signs of the parameter estimates render the interpretation of the outcome at least dubious. These outcomes also support earlier suppositions that additional, theory-based determinants of migration, although extremely helpful in explaining the processes *ex post*, are of very limited use when it comes to forecasting (cf. Kupiszewski, 2002b). The results of the current study seem thus to indicate that demographic factors play a significant role in shaping migration processes, but at the same time they fail to provide more meaningful and precise migration forecasts, very important support of expert judgement notwithstanding. Confirming this notion would, however, require additional studies, exceeding the capacity of IDEA. Nonetheless, several ideas of such research are suggested below.

In order to strengthen the conclusions of the current study, further paths of research might additionally include first of all an extended sensitivity analysis of the forecasts with respect to various changes in the prior distributions. This especially pertains to the VAR models with additional demographic and economic covariates, as it would allow to test various possibilities of the expert knowledge operationalisation and to compare the outcomes with the ones yielded under the non-informative priors. Furthermore, the model classes under study could also be expanded, for example to encompass the structural VAR or other multidimensional models.

For the forecast users, these conclusions imply that migration is indeed uncertain, which is a bad news, but at least the size of this uncertainty can be more or less adequately assessed using the statistical data, additionally enhanced by expert knowledge. The degree of variability of migration processes is itself an important piece of information. The decisions made on the basis of such forecasts, as the ones presented in the current study, will strongly depend on specific problems the decision makers have to face. In particular, they have to assess, what will have more profound consequences: the underestimation or overestimation of future migration flows.

The objectives and constraints of the decision problem, as well as the preferences of the decision makers, will ultimately determine the choice of appropriate quantiles from the predictive distributions – be it upper quartiles, lower 10%, or any other. The median trajectories, well-suited only for problems where an underestimation of future migration would be associated with the same costs as an overestimation by the same amount, may not work well with other decision tasks, or subjective priorities of the decision makers. Irrespective of the selection, the uncertain character of the forecasts has to be always borne in mind. The solution that definitely has to be avoided is just taking the median (or any other) trajectories and treating them as universal, deterministic predictions – they will (almost) surely never come true. In the public sphere, where some caution and degree of moderation is, or should be ideally required whenever the spending of public money is involved, these caveats are especially vital.

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■

Appendix A Data sources for migration flows

B.1. Time series for the total immigration

Table A1.1. Sources of data series for total immigration for the IDEA countries under study

Year	Austria	Czech Republic	France	Hungary	Italy	Poland	Portugal	Spain (*unused*)
1980	:	:	:	:	:	:	:	E
1981	:	:	:	:	E	:	:	E
1982	:	:	:	:	E	:	:	E
1983	:	:	:	:	E	:	:	E
1984	:	:	:	:	E	:	:	E
1985	:	:	:	:	E	:	:	E
1986	:	:	:	:	E	:	:	E
1987	:	:	:	:	E	:	:	E
1988	:	:	:	:	E	:	:	E
1989	:	:	:	:	E	:	:	E
1990	:	:	:	UN	E	E	:	E
1991	:	:	:	UN	E	E	:	E
1992	:	:	:	UN	E	E	E	E
1993	:	UN	:	UN	E	E	E	E
1994	:	UN	INED	UN	E	UN	E	E
1995	:	UN	INED	UN	E	UN	E	E
1996	E	UN	INED	E	E	UN	E	E
1997	E	UN	INED	UN	E	UN	E	E
1998	E	UN	INED	E	CoE	UN	E	E
1999	E	E	INED	E	ISTAT (*)	CoE	INE (**)	E
2000	E	CoE	INED	E	E	CoE	INE (**)	E
2001	E	CoE	INED	E	ISTAT (*)	E	INE (**)	E
2002	E	E	INED	E	E	E	INE (**)	E
2003	E	E	INED	E	E	E	INE (**)	E
2004	E	E	INED	E	ISTAT (*)	E	INE (**)	E
2005	E	E	INED	JMQ	ISTAT (*)	E	INE (**)	E
2006	E	E	:	E (p)	:	E	INE (**)	E
2007	SA	ČSO	:	:	:	GUS	:	:

Notes: E – Eurostat, UN – United Nations Statistics Division, CoE – Council of Europe's Demographic Yearbooks, SA – Statistics Austria, ČSO – Czech Statistical Office, INED – Institut National d'Etudes Démographiques,
JMQ – Joint Migration Questionnaire, ISTAT – demo.istat.it database, GUS – Polish Central Statistical Office, INE – Statistics Portugal, (p) – provisional, (*) by courtesy of IRPPS, (**) by courtesy of SOCIUS

Colon (:) denotes data not available or not reliable, grey shading – own recalculation of original figures

Source: own elaboration.

B.2. Time series for the directions of shares

Table A2.1. Sources of data series for the directions of immigration for the IDEA countries under study

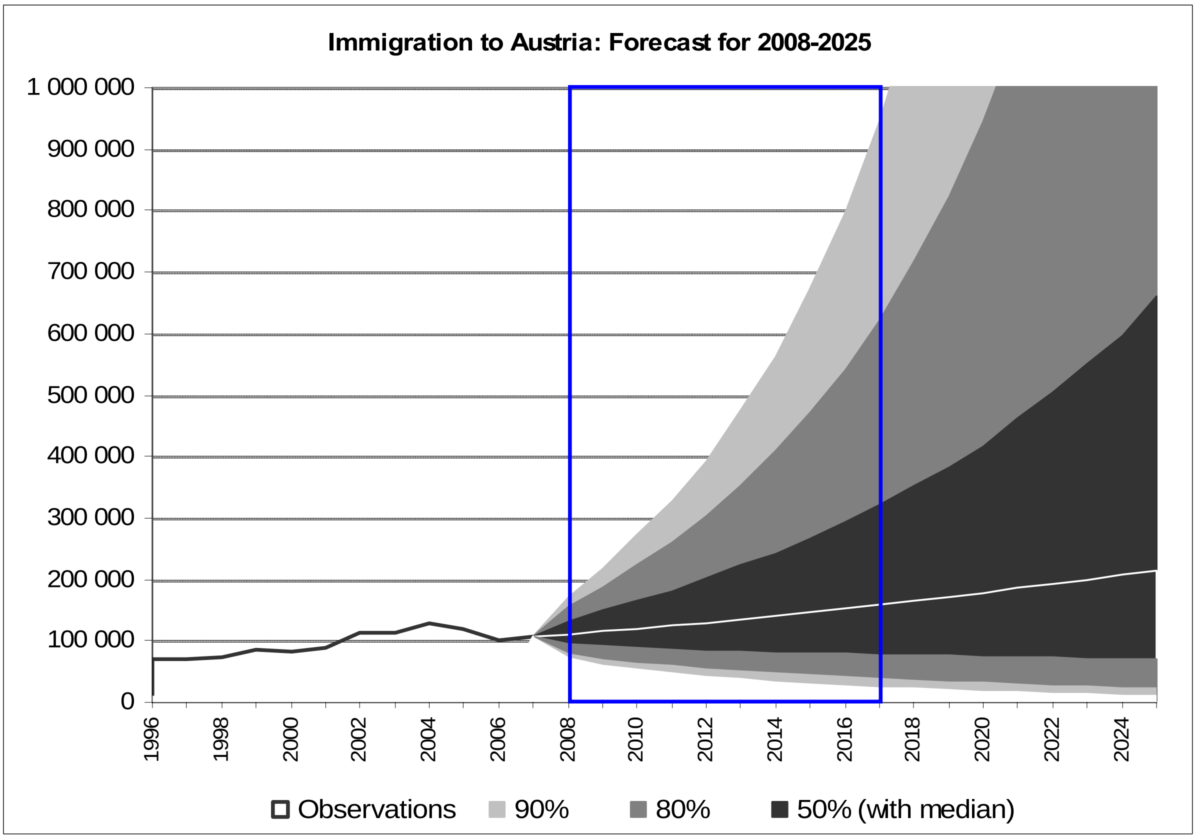
Year	Austria	Czech Republic	France	Hungary	Italy	Poland	Portugal	Spain (*unused*)
1980	:	:	:	:	:	:	:	:
1981	:	:	:	:	:	:	:	:
1982	:	:	:	:	:	:	:	:
1983	:	:	:	:	:	:	:	:
1984	:	:	:	:	:	:	:	:
1985	:	:	:	:	:	:	:	:
1986	:	:	:	:	:	:	:	:
1987	:	:	:	:	:	:	:	:
1988	:	:	:	:	:	:	:	:
1989	:	:	:	:	:	:	:	:
1990	:	:	:	:	:	E / DEM	:	:
1991	:	:	:	:	:	E / DEM	:	:
1992	:	:	:	UN	:	UN	INE (***)	E
1993	:	UN / ČSO (*)	:	UN	:	UN	INE (***)	E
1994	:	E / UN	INED	UN	:	UN	INE (***)	E
1995	:	E / UN	INED	UN	E	UN	INE (***)	E
1996	UN	E / UN	INED	UN	E	UN	INE (***)	E
1997	UN	E / UN	INED	UN	E	UN	INE (***)	E
1998	E	UN / ČSO (*)	INED	E (p)	CoE	UN	INE (***)	E
1999	E	E	INED	E (p)	ISTAT (**)	CoE	INE (***)	E
2000	E	CoE	INED	E (p)	E	CoE	INE (***)	E
2001	E	CoE	INED	E	ISTAT (**)	E	INE (***)	E
2002	E	E	INED	E	E	E	INE (***)	E
2003	E	E	INED	E	E	E	INE (***)	E
2004	E	E	INED	E	E	E	INE (***)	E
2005	E	E	INED	(†)	:	E	INE (***)	E
2006	E	E	:	E	:	E	INE (***)	E
2007	SA	ČSO	:	:	:	GUS (‡)	:	:

Notes: E – Eurostat, UN – United Nations Statistics Division, CoE – Council of Europe's Demographic Yearbooks, SA – Statistics Austria, ČSO – Czech Statistical Office, INED – Institut National d'Etudes Démographiques,
JMQ – Joint Migration Questionnaire, ISTAT – demo.istat.it database, GUS – Polish Central Statistical Office, INE – Statistics Portugal, DEM – *Demografia* (vol. 1991 and 1992), GUS, Warsaw, (p) – provisional, (*) – by courtesy of Charles University, Prague, (**) – by courtesy of IRPPS, (***) – by courtesy of SOCIUS, (†) – *Time series of the international migration, 1990–2000*, HCSO, Budapest; by courtesy of CMRS, (‡) – by courtesy of Ms. Dorota Szałtys.
Colon (:) denotes data not available or not reliable, grey shading – own recalculation of original figures
Source: own elaboration.

Appendix B Detailed results of forecasts

B.3. Austria

Figure B1.1. Immigration to Austria, Random Walk model with constant variance, $p(M_2) = 0.958$



Note: the frame indicates a 10-year forecast horizon (2008–2017)

Source: Data until 2007: Eurostat and Statistics Austria; forecast: own computations.

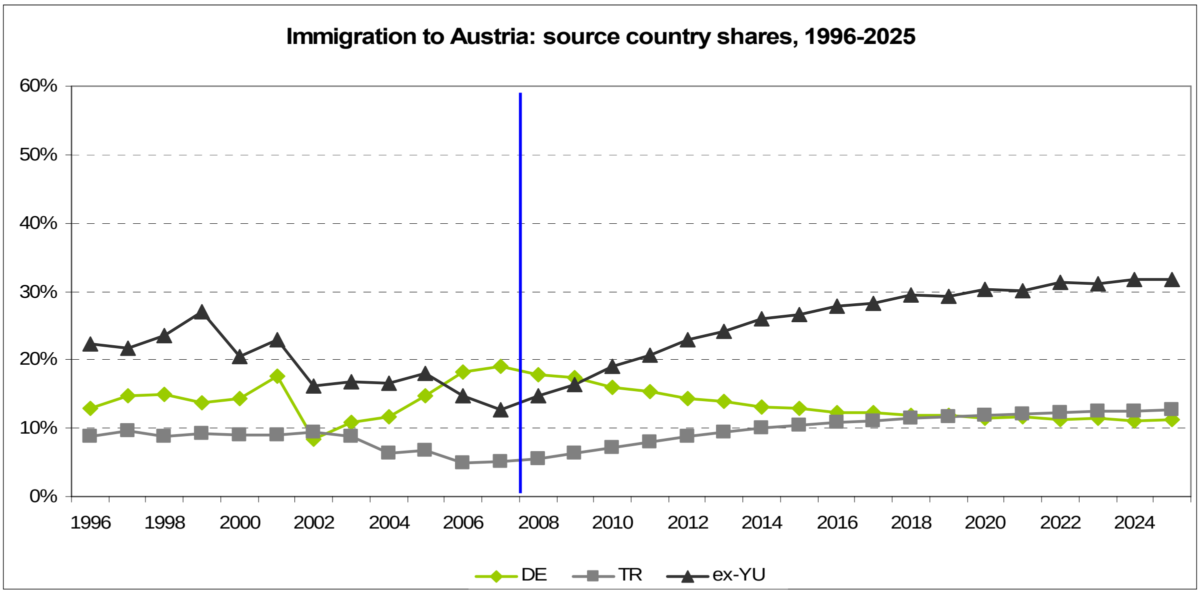
Table B1.1. Summary of predictive distributions for Austria: Median and 50-percent intervals (quartiles)

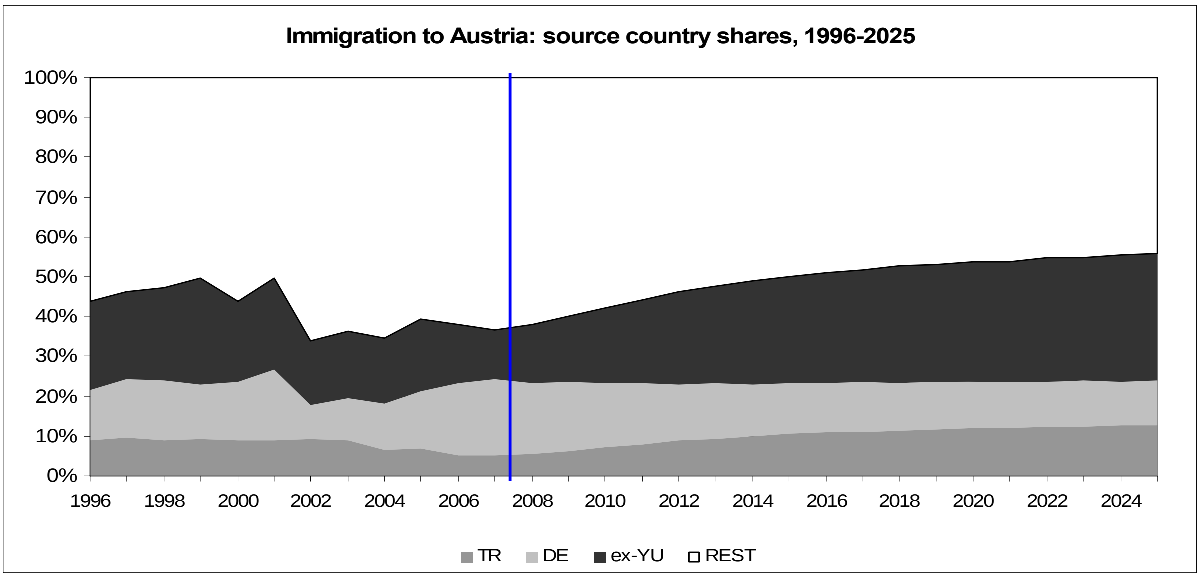
Year	1st Quartile	Median	3rd Quartile
2007 (*)	-	**106 905**	-
2008	93 901	**111 302**	131 926
2009	90 219	**115 844**	148 747
2010	87 553	**120 572**	164 391
2015	80 017	**145 801**	265 667
2020	74 608	**176 310**	416 649
2025	69 564	**215 346**	660 003

(*) 2007 – last observation in the sample

Source: As in Figure B1.1.

Figure B1.2. Immigration to Austria, source country shares, median forecasts (lower graph: cumulative)





Note: Data until 2007, forecast for 2008–2025. Codes: DE – Germany, TR – Turkey, ex-YU – former Yugoslavia
Source: As in Figure B1.1.

Table B1.2. Immigration to Austria, source country shares, median forecasts

Year	Total	DE	TR	ex-YU	Rest
2007 (*)	106 905	20 414	5 412	13 513	67 566
2008	111 302	19 800	6 127	16 406	68 969
2009	115 844	20 279	7 261	18 927	69 378
2010	120 572	19 242	8 672	23 036	69 622
2015	145 801	18 788	15 290	38 668	73 055
2020	176 310	20 382	21 045	53 377	81 506
2025	215 346	24 229	27 290	68 384	95 443

(*) 2007 – last observation in the sample. Codes: DE – Germany, TR – Turkey, ex-YU – former Yugoslavia
Source: As in Figure B1.1.

Table B1.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	1	0	0.1	0	1	0	0.1
ϕ_i	Normal (*a*,*b*)	0.3	6.25	-	-	0.3	6.25	-	-
γ_i	Normal (*a*,*b*)	0.5	1	-	-	0.5	1	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.3202	2	0.3202	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B1.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 9 \\ 1 & 1 & 4 \\ 9 & 4 & 1 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$	3
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} - \\ 0.5 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} - \\ 1 \\ 1 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B1.3.

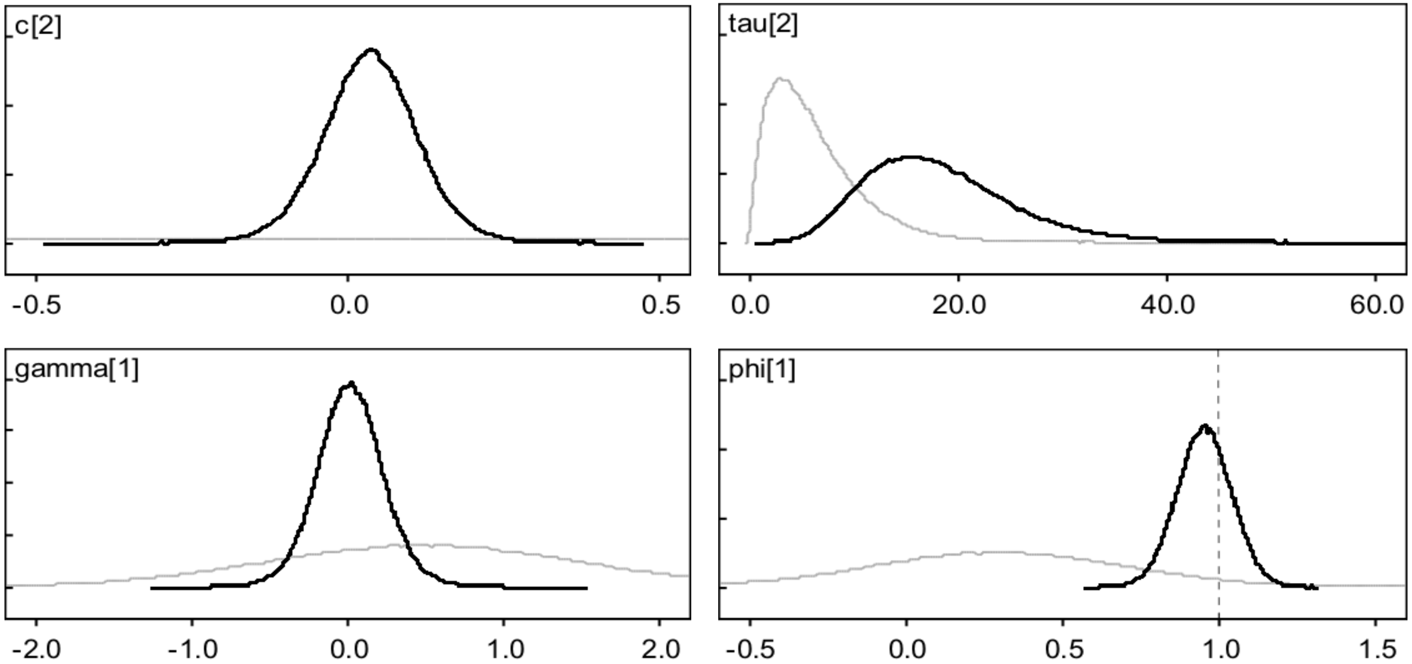
Table B1.5. Prior and posterior probabilities for various models ($M_1 – M_4$)

What did the experts expect? (*a priori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**39.0%**	**21.0%**
Random walk	**26.0%**	**14.0%**

How the data changed it? (*a posteriori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**4.2%**	**0.0%**
Random walk	**95.8%**	**0.0%**

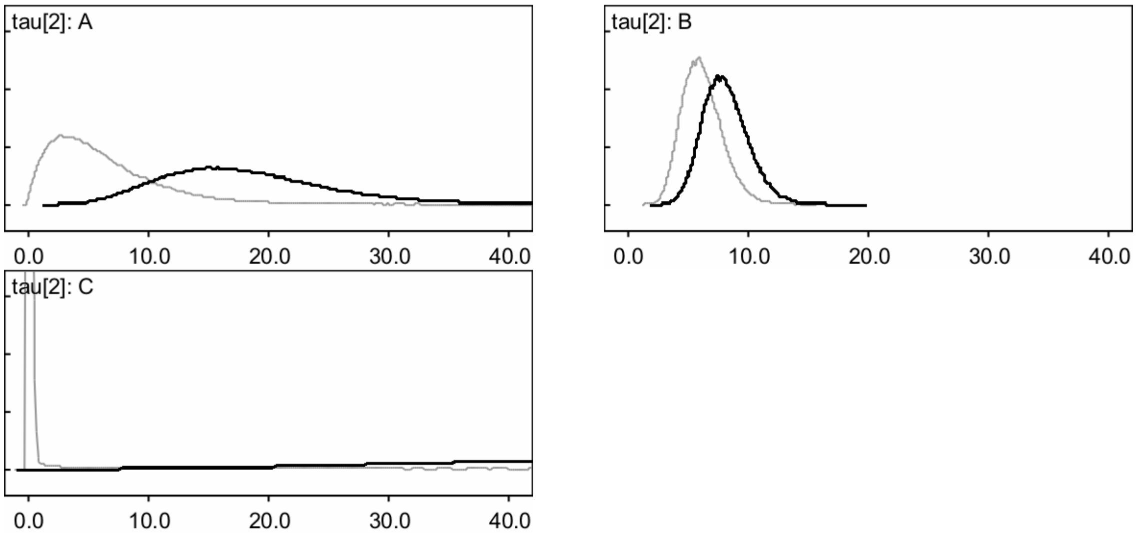
Source: Own elaboration in WinBUGS 3.0.3.

Figure B1.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

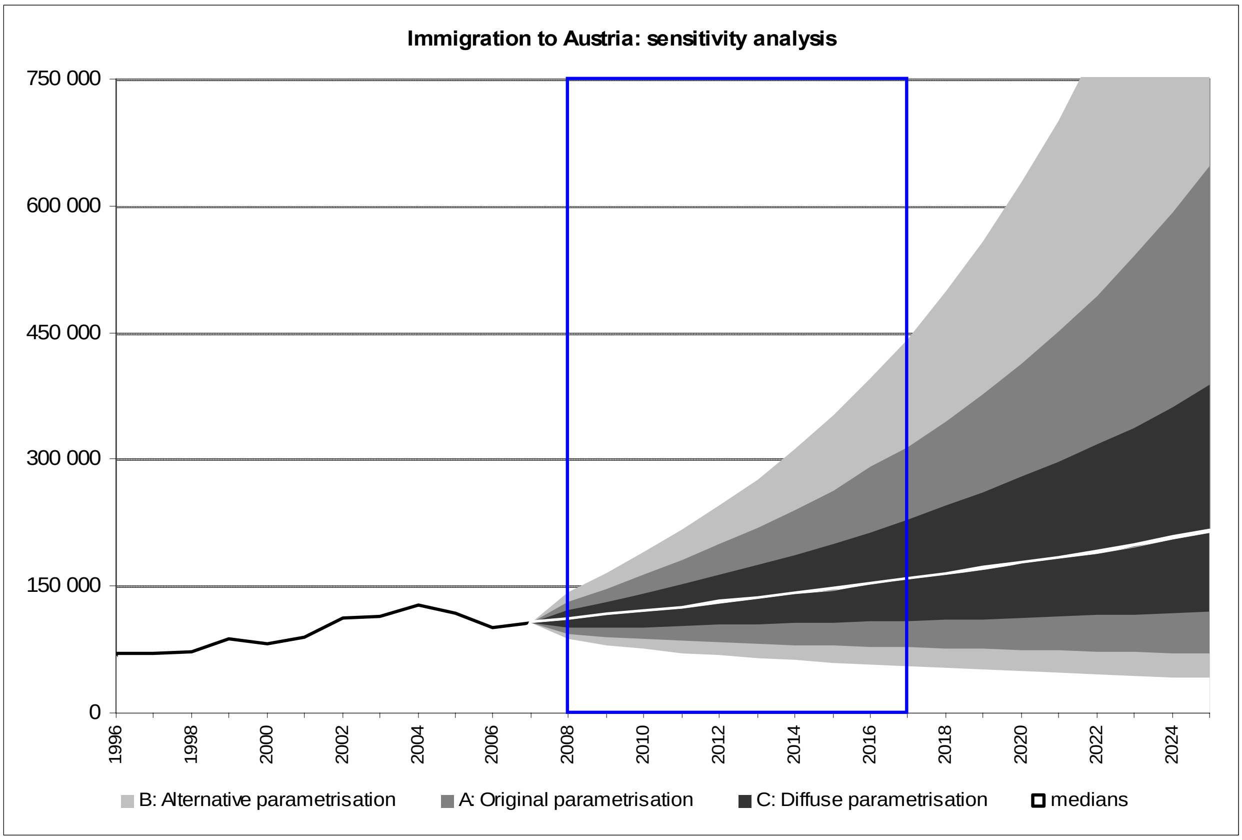
Figure B1.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.3202), B – alternative Γ(14.45, 2.3132), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B1.5. Immigration to Austria, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.3202), B – alternative Γ(14.35, 2.3132), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2008–2017).

Source: Data until 2007: Eurostat and Statistics Austria; forecast: own computations.

Table B1.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Paramete r	Distributio n	Economic model		Demographic model	
		a	*b*	*A*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.3 & 0.92 & -1.1\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}6.25 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.3 & -0.42 & -0.75\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}6.25 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}0.83 & 0.66 & -0.79\\0.66 & 3 & 0\\-0.79 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}0.63 & -0.34 & -0.57\\-0.34 & 3 & 0\\-0.57 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B1.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

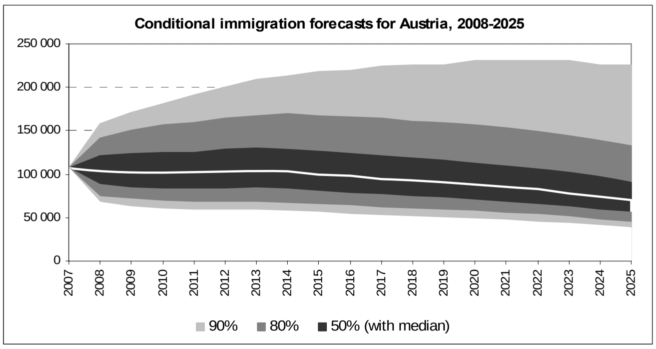
Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.029**	2.579	3.924	**1.009**	2.700	3.838
2nd (Lag)	**1.368**	2.610	3.902	**2.741**	2.693	3.838
Both (Lag)	**1.409**	4.599	6.465	**3.647**	4.605	5.990
1st (Inst)	**0.162**	2.609	4.005	**0.677**	2.646	3.933
2nd (Inst)	**1.467**	2.608	3.944	**1.509**	2.663	3.962
Both (Inst)	**1.719**	4.676	6.618	**2.174**	4.675	6.440

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

Table B1.8, Figure B1.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

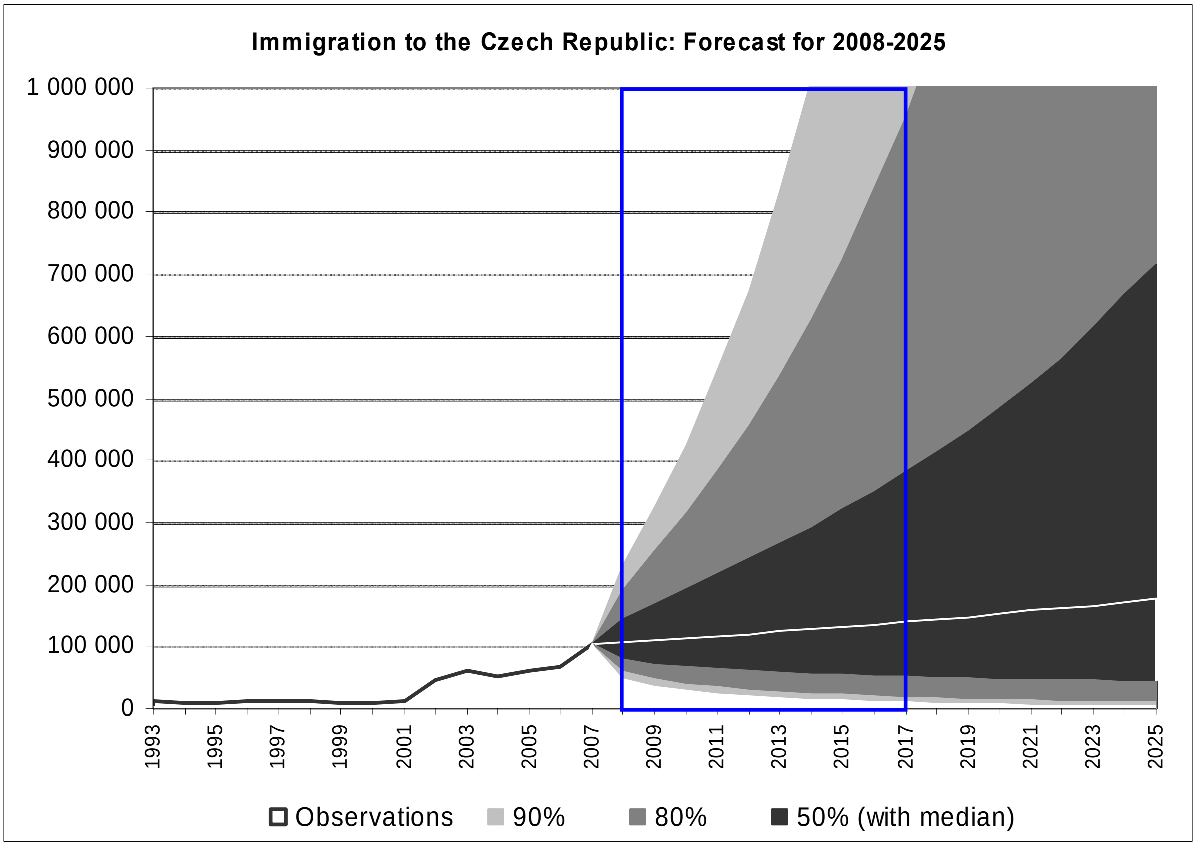
Year	Demographic model (reduced VAR)		
	1st Quartile	Median	3rd Quartile
2008	87 553	102 744	121 783
2009	84 120	101 722	124 244
2010	83 283	101 722	125 492
2015	80 822	99 708	126 754
2020	70 263	87 553	112 420
2025	55 826	70 263	91 126



Notes: The last observation (2007) indicated 106 905 immigrants to Austria. For scenarios (Eurostat/Europop 2008), see report. The figures do not include the predictive uncertainty of the share of population aged 15–64 (2nd variable).
Source: Eurostat, Statistics Austria. Forecast: own computations.

B.4. Czech Republic

Figure B2.1. Immigration to the Czech Republic, Random Walk model with constant variance, $p(M_2) = 0.999$



Note: the frame indicates a 10-year forecast horizon (2008–2017)

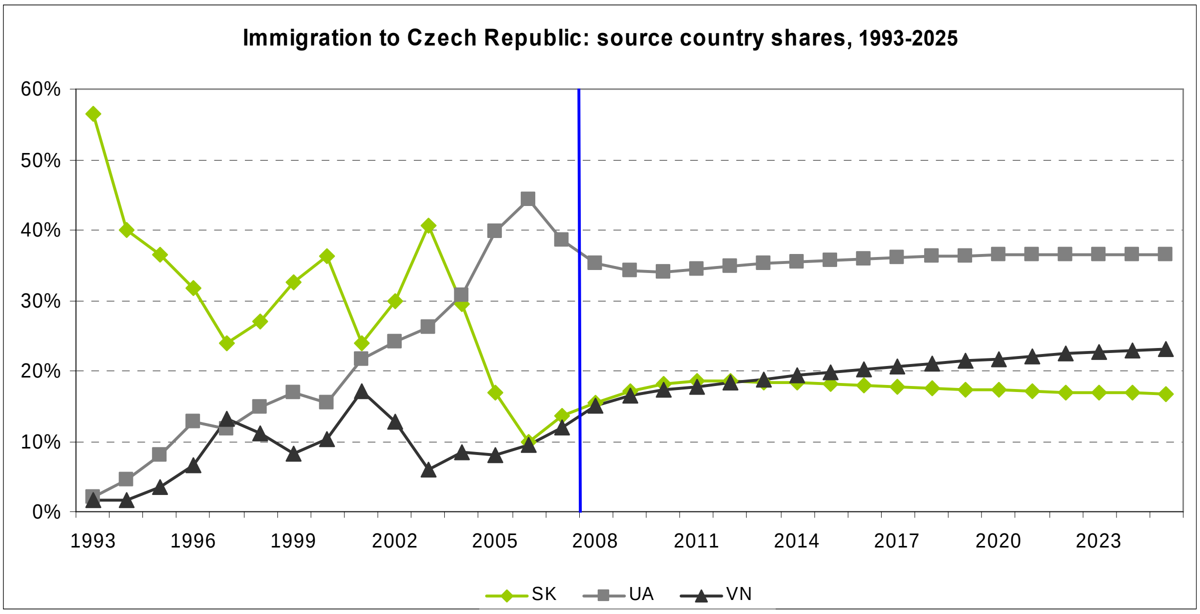
Source: Data until 2007: Eurostat and Czech Statistical Office; forecast: own computations.

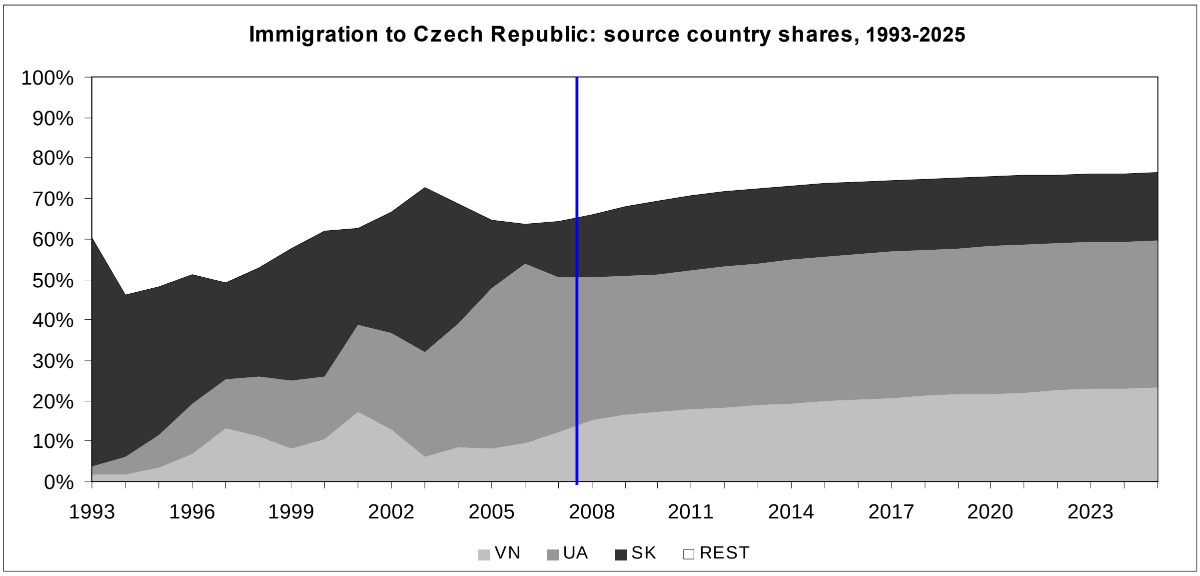
Table B2.1. Summary of predictive distributions for the Czech Republic: Median and 50-percent intervals (quartiles)

Year	1st Quartile	Median	3rd Quartile
2007 (*)	-	**104 445**	-
2008	79 221	**106 938**	144 351
2009	71 682	**110 194**	169 397
2010	66 836	**113 550**	192 914
2015	53 637	**131 926**	321 258
2020	47 099	**153 277**	484 077
2025	42 193	**176 310**	714 973

(*) 2007 – last observation in the sample
Source: As in Figure B2.1.

Figure B2.2. Immigration to the Czech Republic, source country shares, median forecasts (lower graph: cumulative)





Note: Data until 2007, forecast for 2008–2025. Codes: SK – Slovakia, UA – Ukraine, VN – Vietnam
Source: As in Figure B2.1.

Table B2.2. Immigration to the Czech Republic, source country shares, median forecasts

Year	Total	SK	UA	VN	Rest
2007 (*)	104 445	14 194	40 319	12 565	37 368
2008	106 938	16 617	37 700	16 200	36 420
2009	110 194	18 893	37 752	18 149	35 400
2010	113 550	20 712	38 668	19 568	34 602
2015	131 926	23 954	47 065	26 139	34 769
2020	153 277	26 457	55 981	33 156	37 683
2025	176 310	29 613	64 322	40 838	41 538

(*) 2007 – last observation in the sample. Codes: SK – Slovakia, UA – Ukraine, VN – Vietnam
Source: As in Figure B2.1.

Table B2.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	0.001	0	400	0	0.001	0	400
ϕ_i	Normal (*a*,*b*)	0.396	1.778	-	-	0.396	1.778	-	-
γ_i	Normal (*a*,*b*)	0	0.0001	-	-	0	0.0001	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.5254	2	0.5254	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1
b_i (dummy)	Normal (a,b)	-	-	–1	1	-	-	–1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B2.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$	3
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} - \\ - \\ 1 \end{pmatrix}$	$\begin{pmatrix} - \\ - \\ 1 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B2.3.

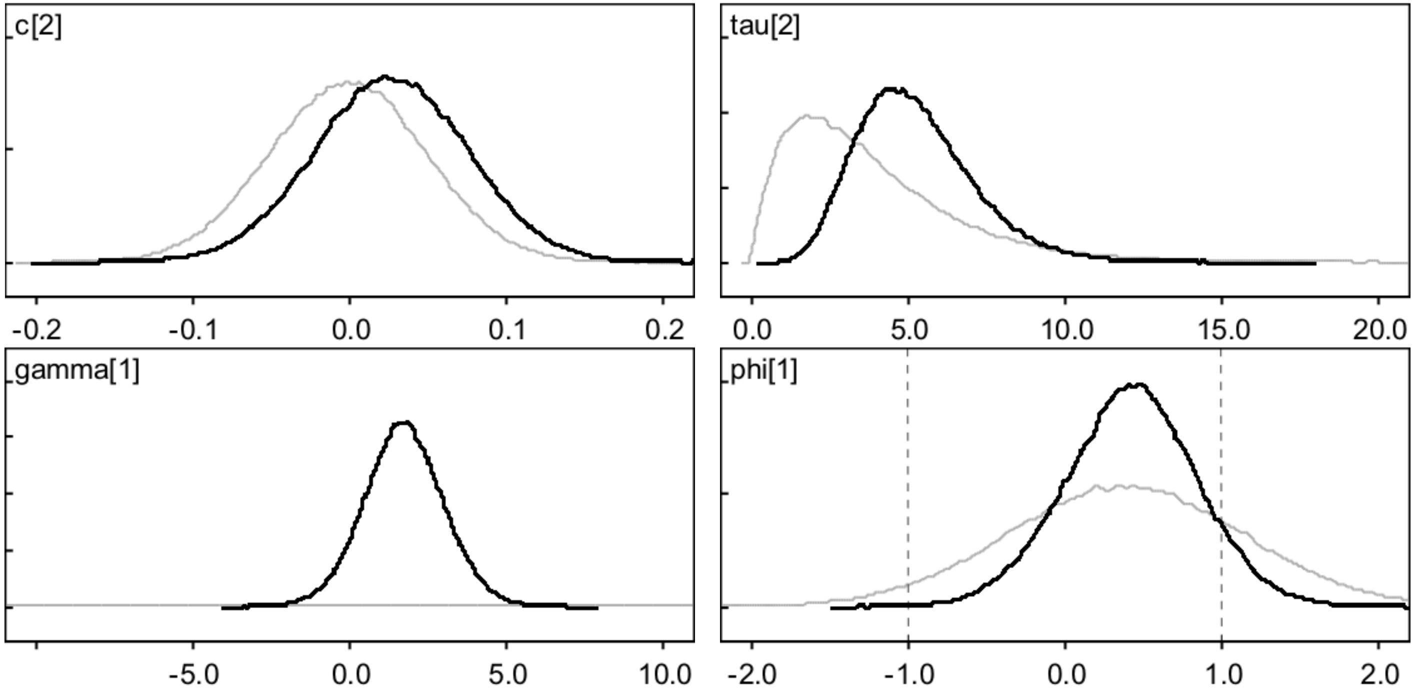
Table B2.5. Prior and posterior probabilities for various models ($M_1 - M_4$)

What did the experts expect? (*a priori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**43.3%**	**23.3%**
Random walk	**21.7%**	**11.7%**

How the data changed it? (*a posteriori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**0.1%**	**0.0%**
Random walk	**99.9%**	**0.0%**

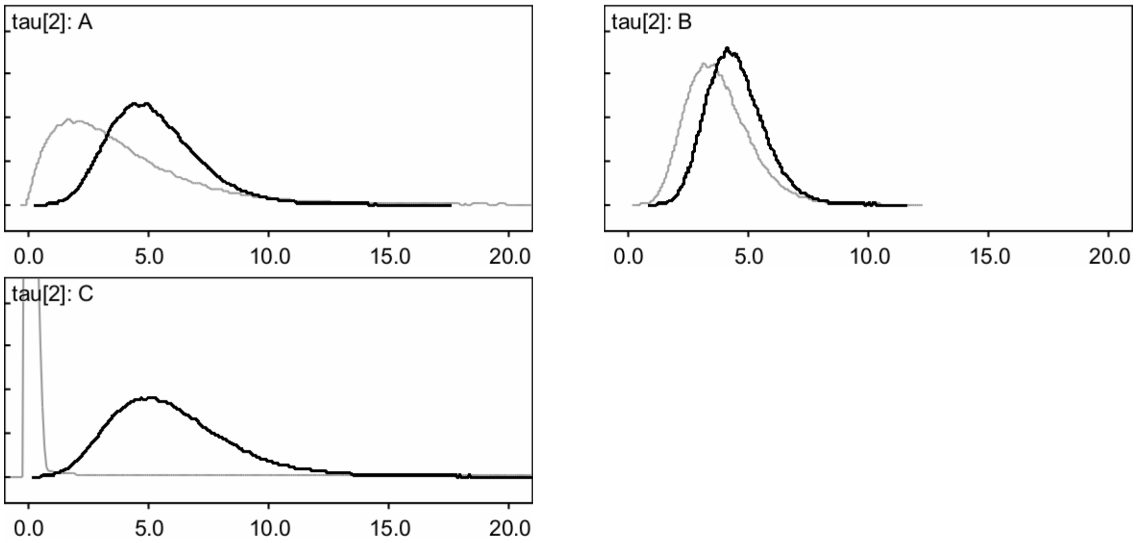
Source: Own elaboration in WinBUGS 3.0.3.

Figure B2.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

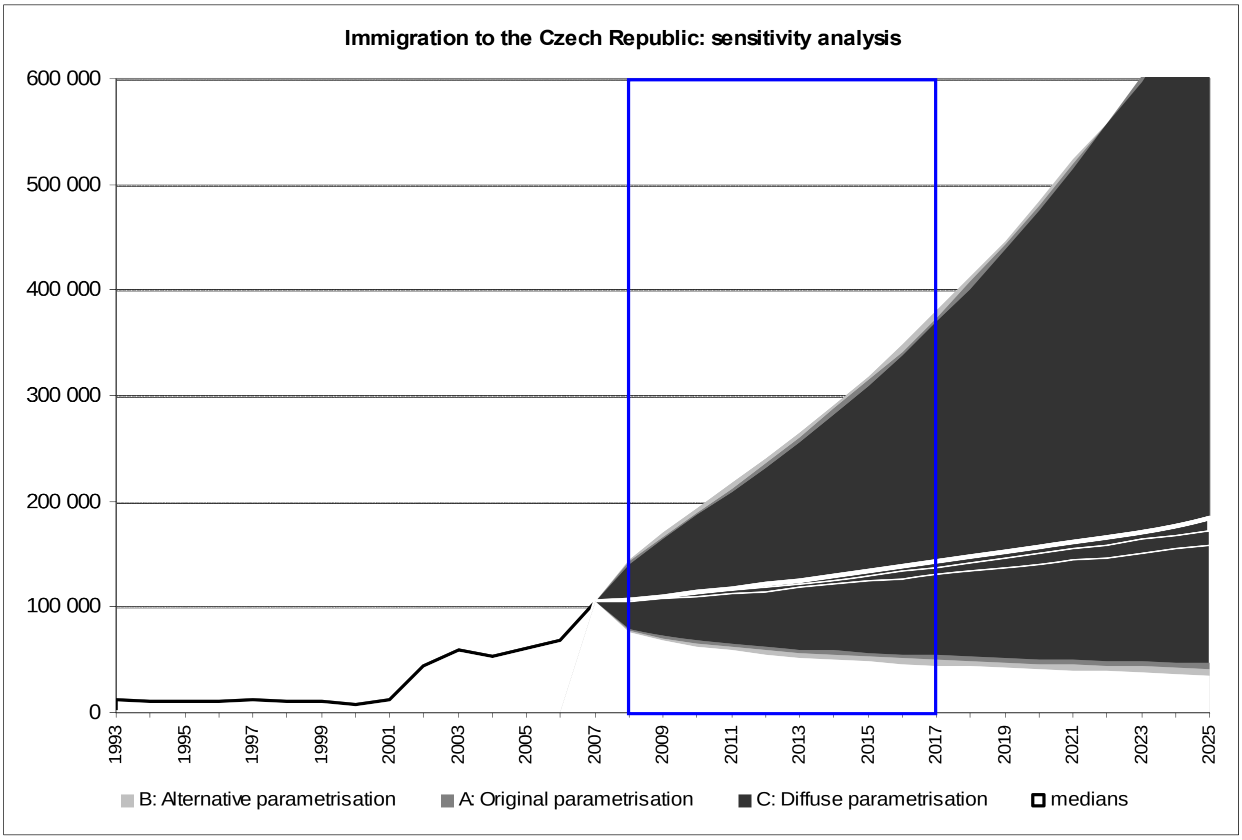
Figure B2.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.5254), B – alternative Γ(8.5201, 2.2383), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B2.5. Immigration to the Czech Republic, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.5254), B – alternative Γ(8.5201, 2.2383), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2008–2017).

Source: Data until 2007: Eurostat and Czech Statistical Office; forecast: own computations.

Table B2.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model		Demographic model	
		a	*b*	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.4 & 1.29 & -1.31\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.78 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.4 & -0.81 & -1.31\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.78 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}2.76 & 2.04 & -1.32\\2.04 & 3 & 0\\-1.32 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}1.24 & -0.52 & -1.04\\-0.52 & 3 & 0\\-1.04 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: Own elaboration, partially based on the Delphi expert survey.

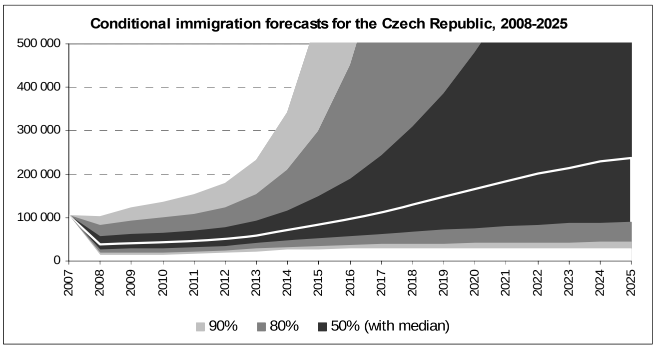
Table B2.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.711**	2.587	3.894	**2.905**	2.704	3.863
2nd (Lag)	**0.232**	2.596	3.925	**0.993**	2.610	3.866
Both (Lag)	**0.908**	4.593	6.447	**3.497**	4.590	6.152
1st (Inst)	**0.006**	2.640	3.948	**0.710**	2.637	3.998
2nd (Inst)	**0.653**	2.616	3.951	**2.942**	2.639	3.996
Both (Inst)	**0.741**	4.657	6.522	**3.637**	4.698	6.587

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.
Source: Own elaboration in WinBUGS 3.0.3.

Table B2.8, Figure B2.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

Year	Demographic model (general VAR)		
	1st Quartile	Median	3rd Quartile
2008	26 108	38 561	56 387
2009	26 903	40 538	61 084
2010	27 723	42 193	64 216
2015	51 534	83 283	147 267
2020	74 608	166 043	479 261
2025	90 219	237 994	936 589

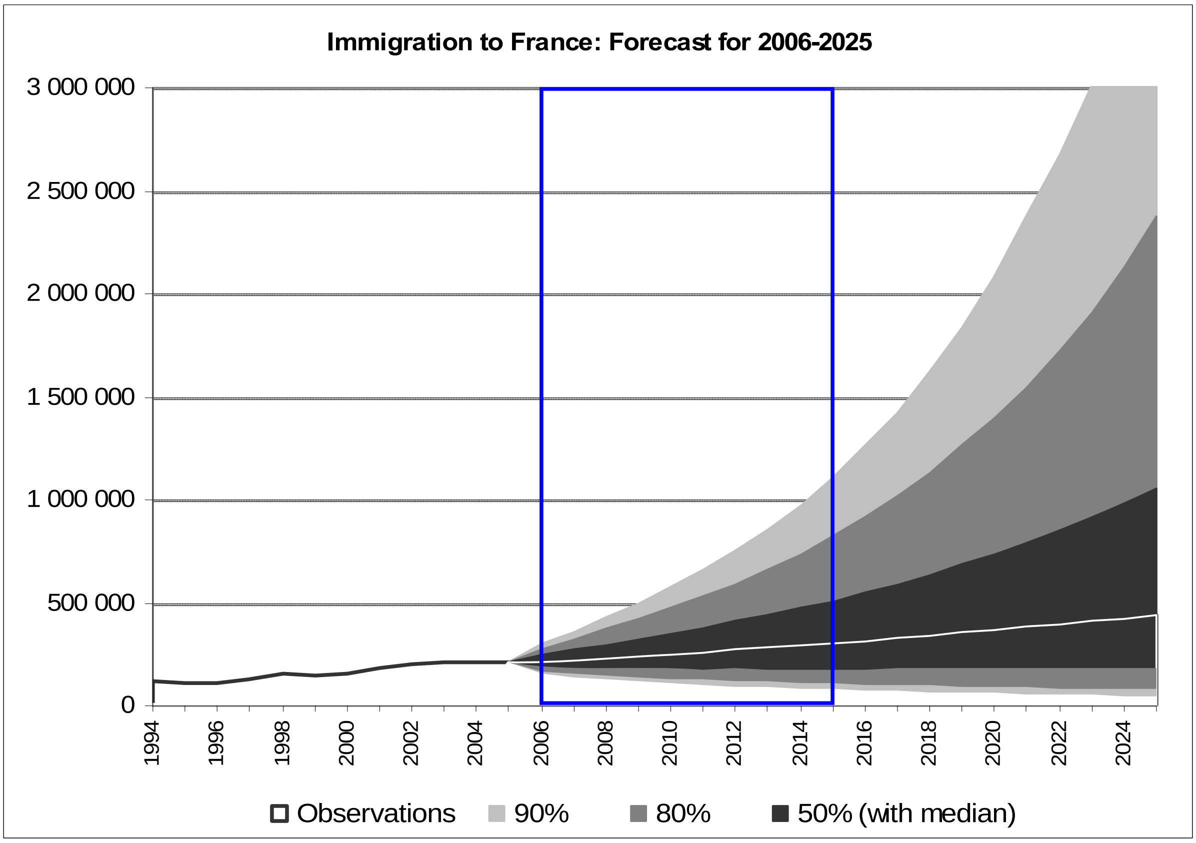


Notes: The last observation (2007) indicated 104 445 immigrants to the Czech Republic. For scenarios (Eurostat/ Europop 2008), see report. The figures do not include the predictive uncertainty of two additional demographic variables.

Source: Eurostat, Czech Statistical Office. Forecast: own computations.

B.5. France

Figure B3.1. Immigration to France, Random Walk model with constant variance, $p(M_2) = 0.998$



Note: the frame indicates a 10-year forecast horizon (2006–2015)

Source: Data until 2005 obtained by courtesy of INED (Institut National d'Etudes Démographiques).

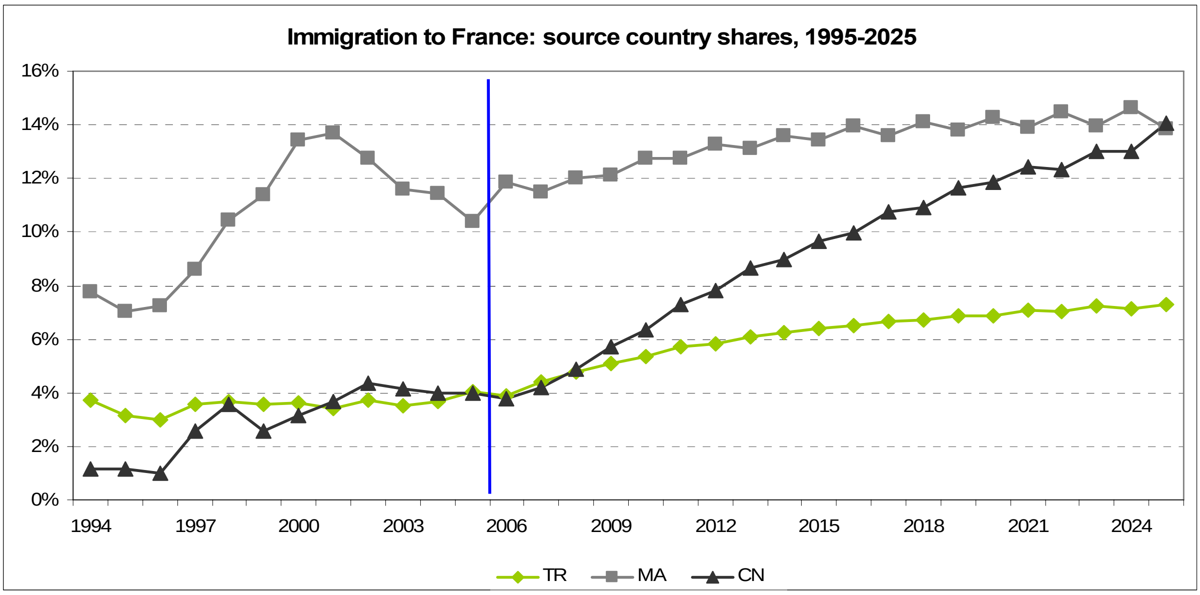
Table B3.1. Summary of predictive distributions for France: Median and 50-percent intervals (quartiles)

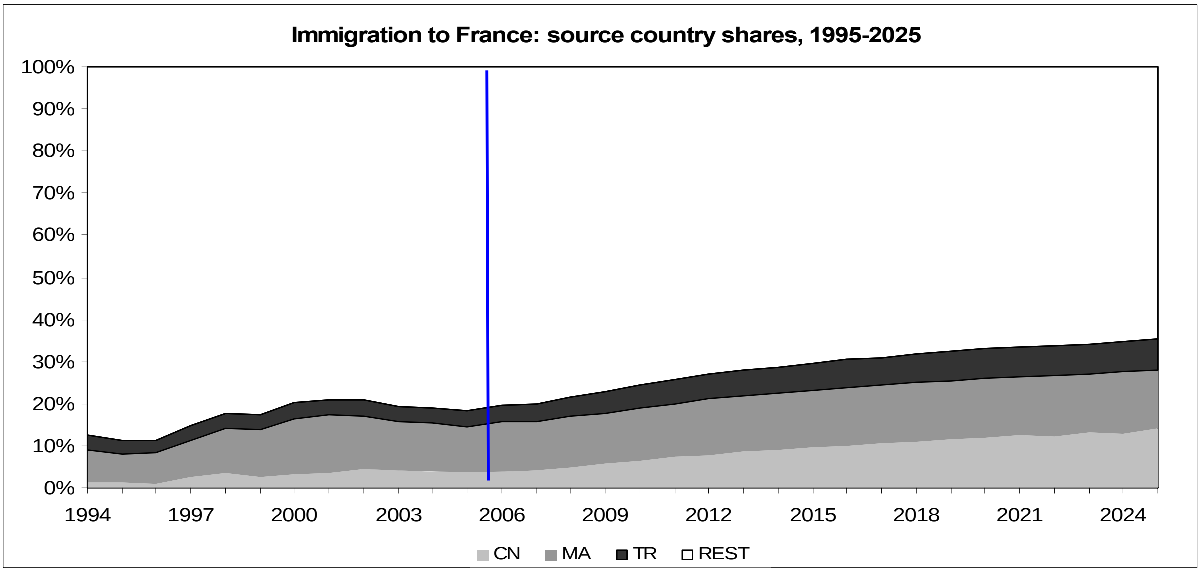
Year	1st Quartile	Median	3rd Quartile
2005 (*)	-	**207 562**	-
2007	185 350	**224 134**	271 034
2008	181 680	**233 281**	269 559
2010	179 872	**250 196**	348 015
2015	178 082	**302 549**	508 897
2020	181 680	**365 858**	736 747
2025	183 506	**442 413**	1 056 001

(*) 2005 – last observation in the sample

Source: As in Figure B3.1.

Figure B3.2. Immigration to France, source country shares (citizenship), median forecasts (lower graph: cumulative)





Note: Data until 2005, forecast for 2006–2025. Codes: CN – China, MA – Morocco, TR – Turkey
Source: As in Figure B3.1.

Table B3.2. Immigration to France, source country shares (citizenship), median forecasts

Year	Total	TR	MA	CN	Rest
2005 (*)	207 562	8 342	21 580	8 221	169 419
2007	224 134	9 884	25 762	9 393	179 095
2008	233 281	11 080	28 082	11 372	182 749
2010	250 196	13 432	31 932	15 857	188 975
2015	302 549	19 385	40 670	29 239	213 256
2020	365 858	25 129	52 249	43 294	245 186
2025	442 413	32 207	61 387	62 190	286 630

(*) 2005 – last observation in the sample. Codes: CN – China, MA – Morocco, TR – Turkey
Source: As in Figure B3.1.

Table B3.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	0.001	0	100	0	0.001	0	100
ϕ_i	Normal (*a*,*b*)	0.5	1.656	-	-	0.5	1.656	-	-
γ_i	Normal (*a*,*b*)	0.5	1	-	-	0.5	1	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.1863	2	0.1863	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B3.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}$
$\phi = [\phi_{ij}]$	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 10 & 10 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	3
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0.5 \\ 0 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5 \\ 0 \\ 0.5 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B3.3.

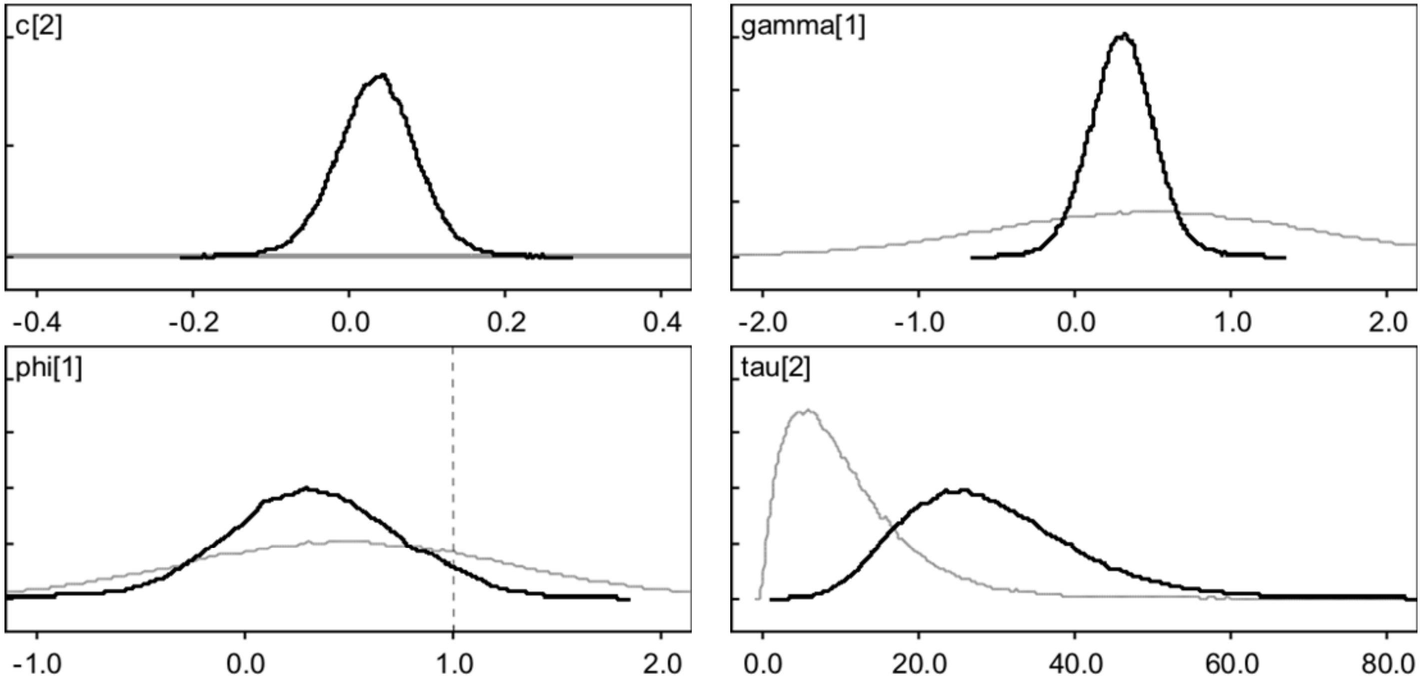
Table B3.5. Prior and posterior probabilities for various models (M_1 – M_4)

What did the experts expect? (*a priori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**30.0%**	**39.0%**
Random walk	**13.5%**	**17.5%**

How the data changed it? (*a posteriori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**0.1%**	**0.0%**
Random walk	**99.8%**	**0.0%**

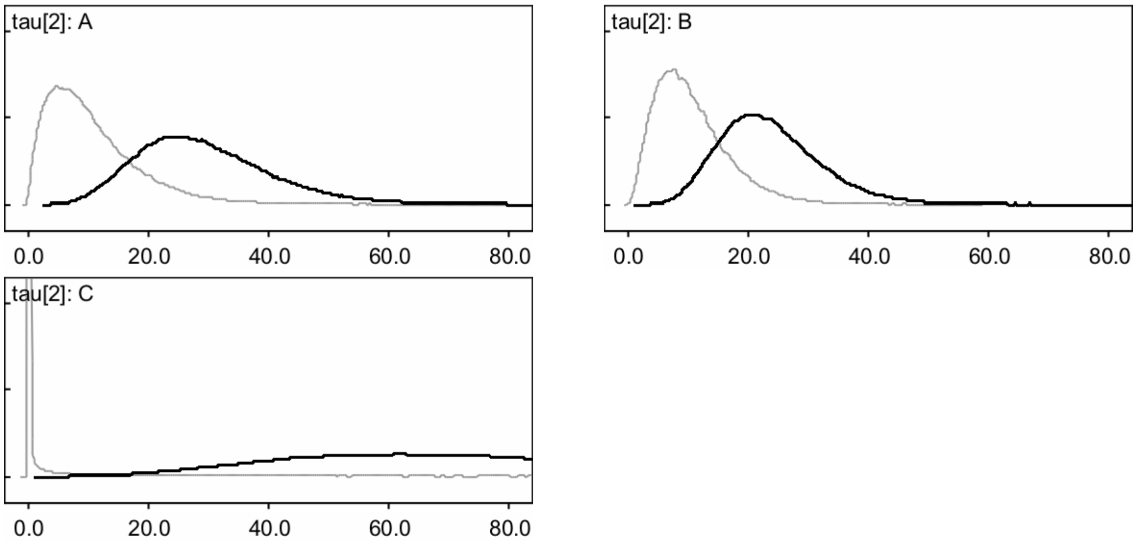
Source: Own elaboration in WinBUGS 3.0.3.

Figure B3.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

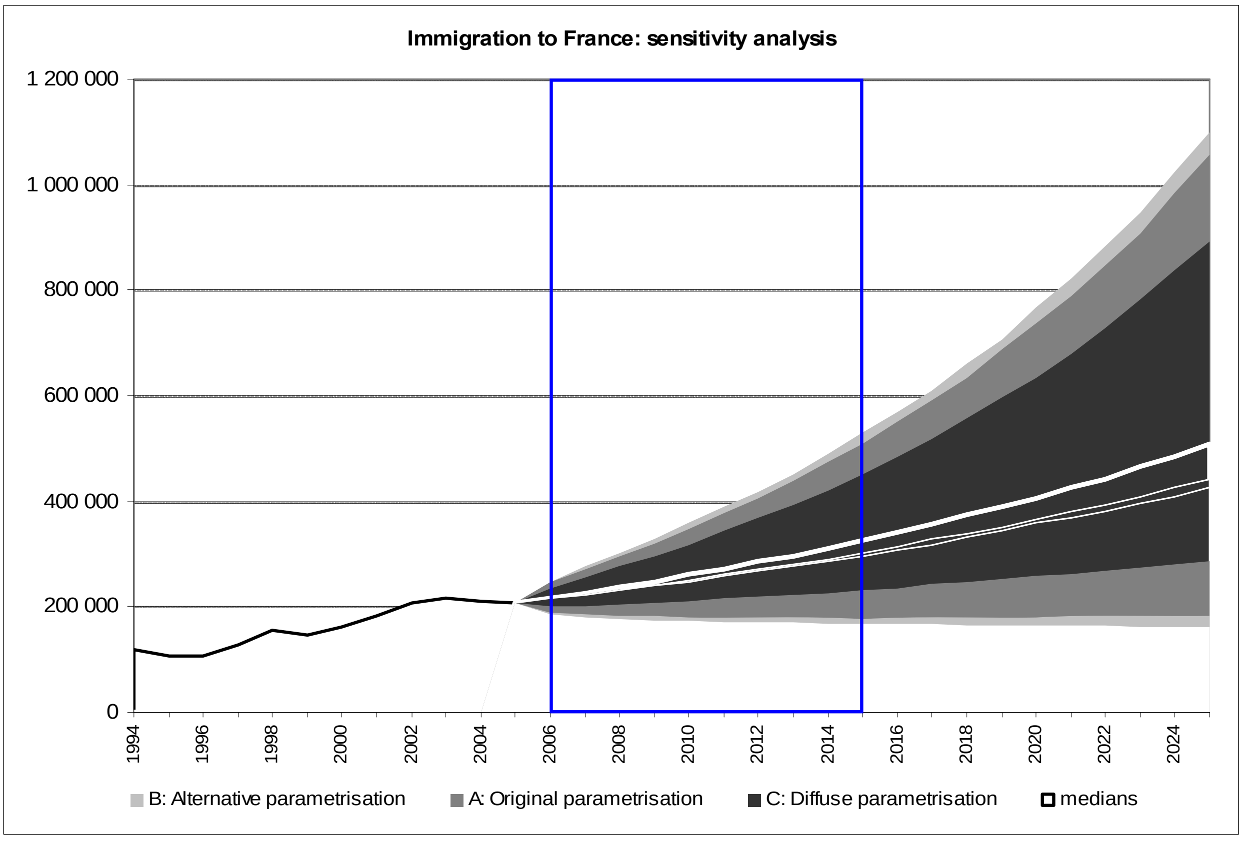
Figure B3.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.1864), B – alternative Γ(3.125, 0.2912), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B3.5. Immigration to France, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.1864), B – alternative Γ(3.125, 0.2912), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2006–2015).

Source: Data until 2005 obtained by courtesy of INED; forecast: own computations.

Table B3.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model		Demographic model	
		a	*b*	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.5 & 1 & -0.19\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.66 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.5 & -1 & -0.19\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.66 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}0.39 & 0.54 & -0.20\\0.54 & 3 & 0\\-0.20 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}1.01 & -0.87 & -1.20\\-0.87 & 3 & 0\\-1.20 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B3.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

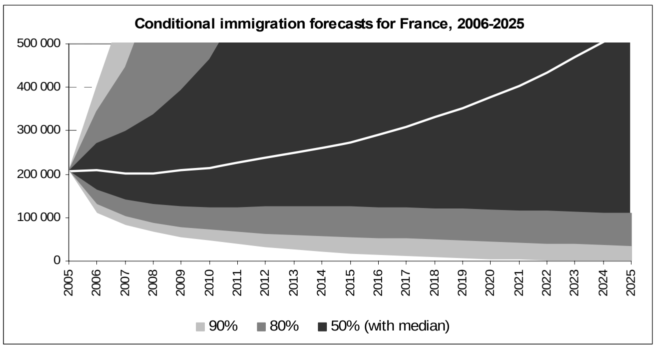
Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.212**	2.571	3.955	**3.517**	2.697	3.817
2nd (Lag)	**1.762**	2.654	3.942	**0.000**	2.765	3.925
Both (Lag)	**1.762**	4.639	6.513	**3.517**	4.637	6.023
1st (Inst)	**3.325**	2.637	3.927	**6.037**	2.644	3.979
2nd (Inst)	**0.122**	2.628	3.976	**11.50**	2.644	3.976
Both (Inst)	**4.067**	4.653	6.512	**17.46**	4.693	6.444

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

Table B3.8, Figure B3.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

Year	Demographic model (general VAR)		
	1st Quartile	Median	3rd Quartile
2006	164 391	208 981	271 034
2008	130 614	202 805	337 729
2010	123 007	215 346	465 096
2015	124 244	273 758	2 002 686
2020	117 008	377 000	Not plausible
2025	109 098	540 365	Not plausible

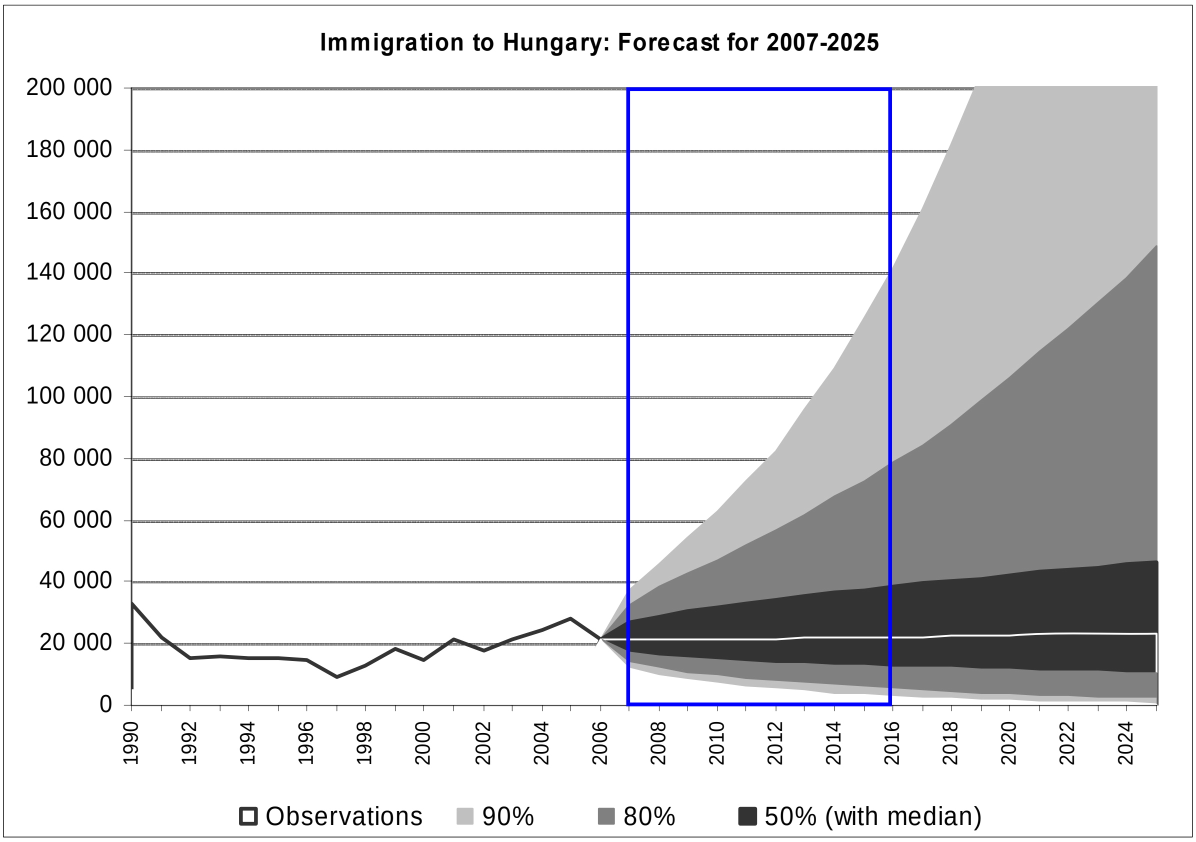


Notes: The last observation (2005) indicated 207 562 immigrants to France. For scenarios (Eurostat/Europop 2008), see report. The figures do not include the predictive uncertainty of two additional demographic variables.

Source: Eurostat, INED. Forecast: own computations.

B.6. Hungary

Figure B4.1. Immigration to Hungary, averaged forecasts from Random Walk and Autoregression models with constant variance, $p(M_2) = 0.502$ and $p(M_1) = 0.498$



Note: the frame indicates a 10-year forecast horizon (2007–2016)

Source: Data until 2006: Eurostat and Hungarian Central Statistical Office; forecast: own computations.

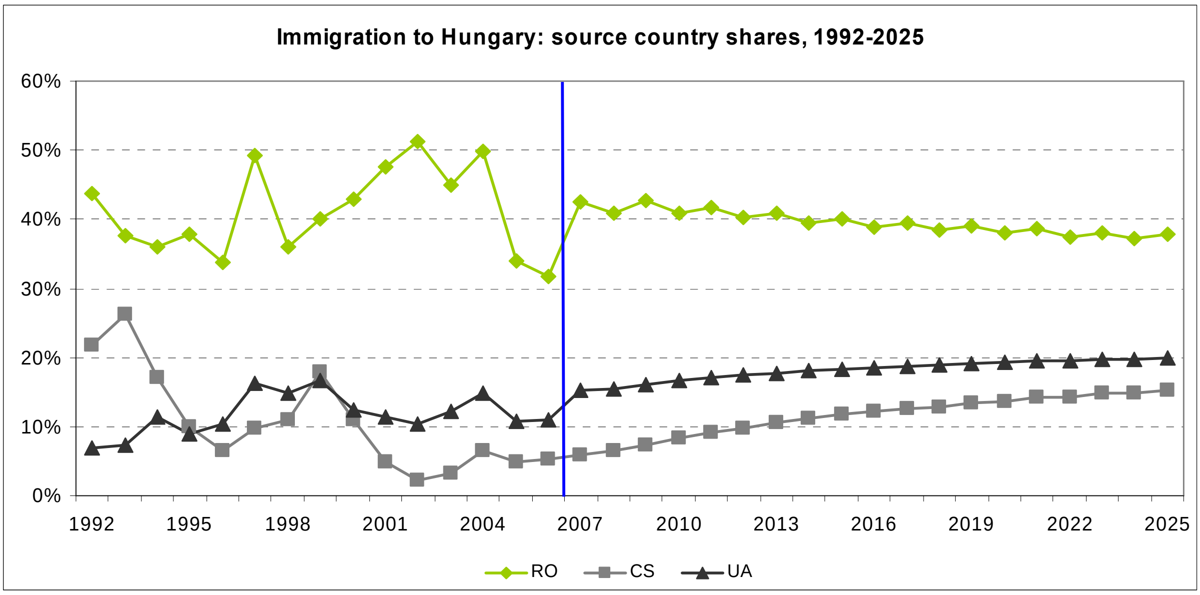
Table B4.1. Summary of predictive distributions for Hungary: Median and 50-percent intervals (quartiles)

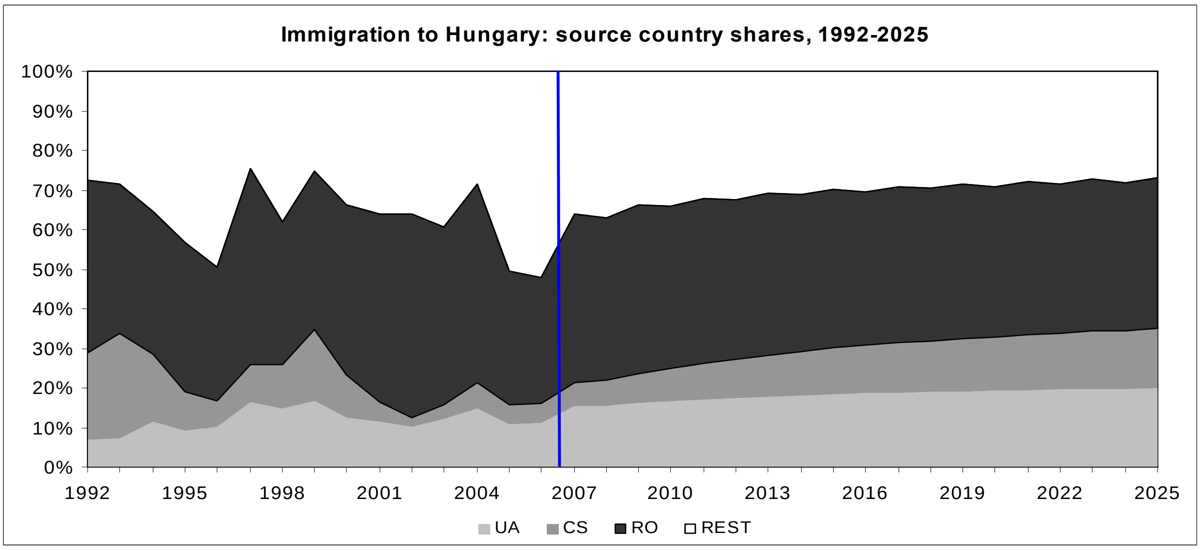
Year	1st Quartile	Median	3rd Quartile
2006 (*)	-	**21 520**	-
2007	17 086	**21 311**	26 635
2008	15 899	**21 354**	28 567
2010	14 574	**21 440**	31 571
2015	12 810	**22 026**	37 049
2020	11 430	**22 697**	42 193
2025	10 280	**23 156**	46 630

(*) 2006 – last observation in the sample

Source: As in Figure B4.1.

Figure B4.2. Immigration to Hungary, source country shares (citizenship), median forecasts (lower graph: cumulative)





Note: Data until 2006, forecast for 2007–2025. Codes: UA – Ukraine, CS – Serbia and Montenegro, RO – Romania
Source: As in Figure B4.1.

Table B4.2. Immigration to Hungary, source country shares (citizenship), median forecasts

Year	Total	RO	CS	UA	Rest
2006 (*)	21 520	6 813	1 120	2 365	11 222
2007	21 311	9 071	1 266	3 257	7 718
2008	21 354	8 729	1 410	3 313	7 903
2010	21 440	8 766	1 772	3 573	7 329
2015	22 026	8 831	2 586	4 042	6 568
2020	22 697	8 637	3 085	4 376	6 600
2025	23 156	8 752	3 549	4 599	6 257

(*) 2006 – last observation in the sample. Codes: RO – Romania, CS – Serbia and Montenegro, UA – Ukraine
Source: As in Figure B4.1.

Table B4.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	0.001	0	0.01	0	0.001	0	0.01
ϕ_i	Normal (*a*,*b*)	0.3	0.25	-	-	0.3	0.25	-	-
γ_i	Normal (*a*,*b*)	0.5	1	-	-	0.5	1	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.4127	2	0.4127	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B4.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 & 1 & 1 \\ 1 & 0.1 & 1 \\ 1 & 1 & 0.1 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	3
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0.1 \\ 0.5 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B4.3.

Table B4.5. Prior and posterior probabilities for various models ($M_1 - M_4$)

What did the experts expect? (*a priori*)

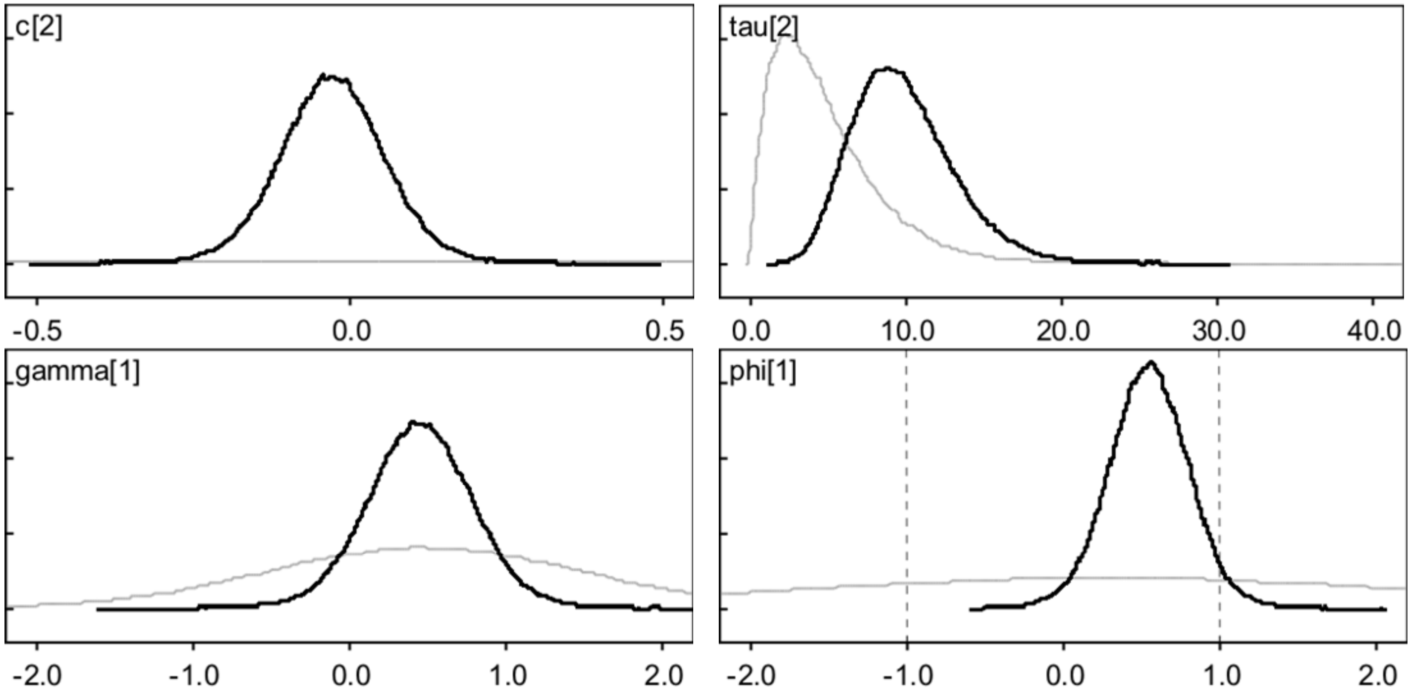
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**39.2%**	**43.3%**
Random walk	**8.3%**	**9.2%**

How the data changed it? (*a posteriori*)

Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**49.8%**	**0.0%**
Random walk	**50.2%**	**0.0%**

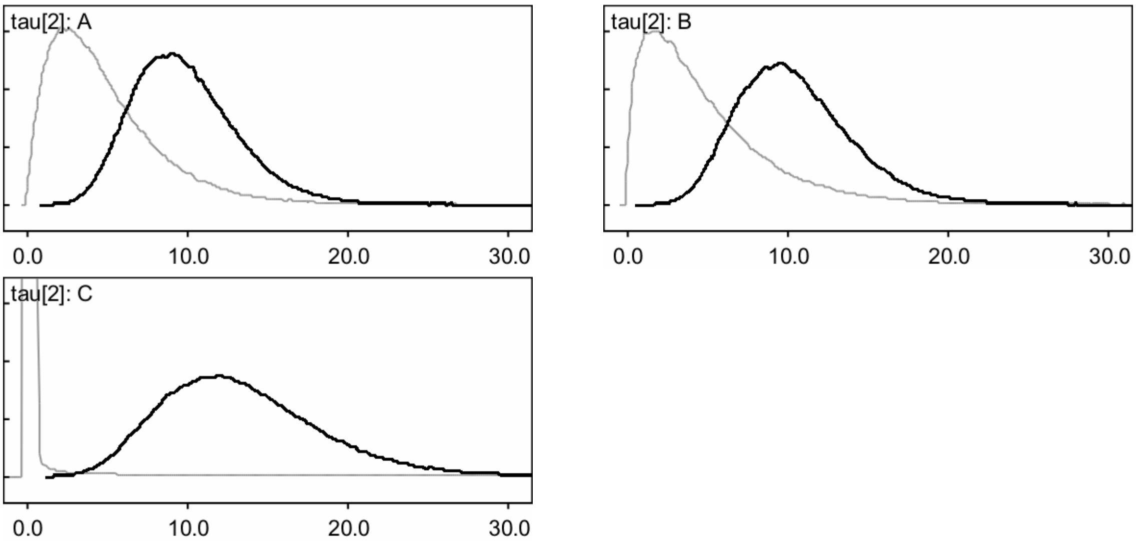
Source: Own elaboration in WinBUGS 3.0.3.

Figure B4.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

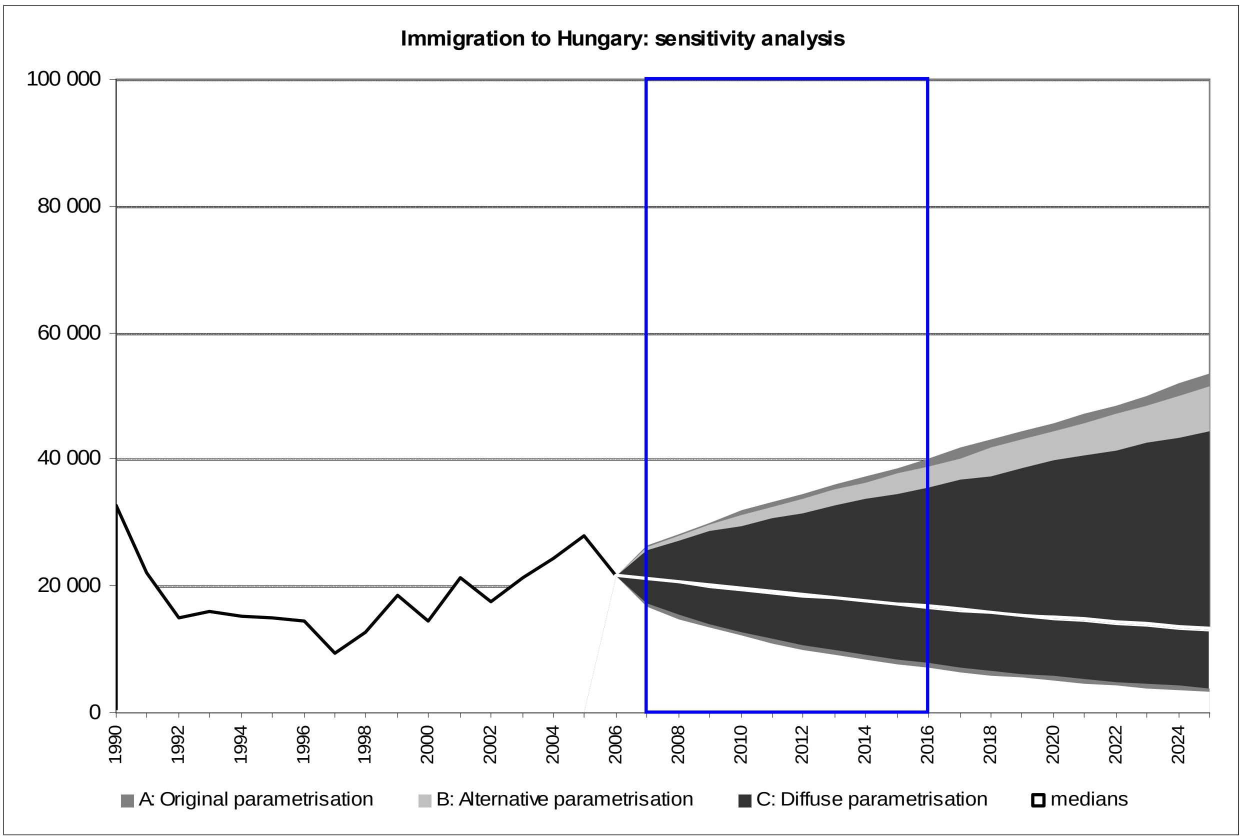
Figure B4.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.4127), B – alternative Γ(1.5046, 0.3105), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B4.5. Immigration to Hungary, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.4127), B – alternative Γ(1.5046, 0.3105), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2007–2016).

Source: Data until 2006: Eurostat and Hungarian Central Statistical Office; forecast: own computations.

Table B4.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model *a*	Economic model *b*	Demographic model *a*	Demographic model *b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.3 & 1.25 & -0.67\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}0.25 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.3 & -0.83 & -1.42\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}4 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}1.90 & 1.79 & -0.80\\1.79 & 3 & 0\\-0.80 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}1.13 & -0.61 & -1.07\\-0.61 & 3 & 0\\-1.07 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B4.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

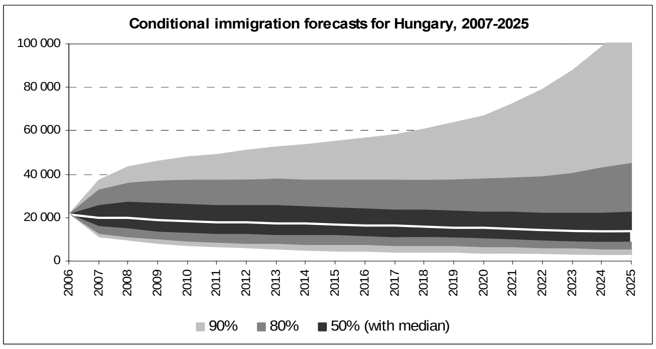
Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.543**	2.660	3.917	**2.669**	2.701	3.848
2nd (Lag)	**0.496**	2.663	3.909	**0.527**	2.710	3.805
Both (Lag)	**1.611**	4.643	6.366	**4.060**	4.595	6.001
1st (Inst)	**0.151**	2.654	3.952	**1.505**	2.682	3.925
2nd (Inst)	**0.015**	2.662	3.954	**4.360**	2.685	3.932
Both (Inst)	**0.219**	4.692	6.378	**5.857**	4.665	6.326

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

Table B4.8, Figure B4.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

Year	Demographic model (general VAR)		
	1st Quartile	Median	3rd Quartile
2007	15 615	20 010	25 591
2009	13 507	18 883	26 370
2010	12 874	18 343	25 848
2015	11 499	16 933	24 588
2020	10 158	15 123	22 697
2025	8 578	13 602	22 248

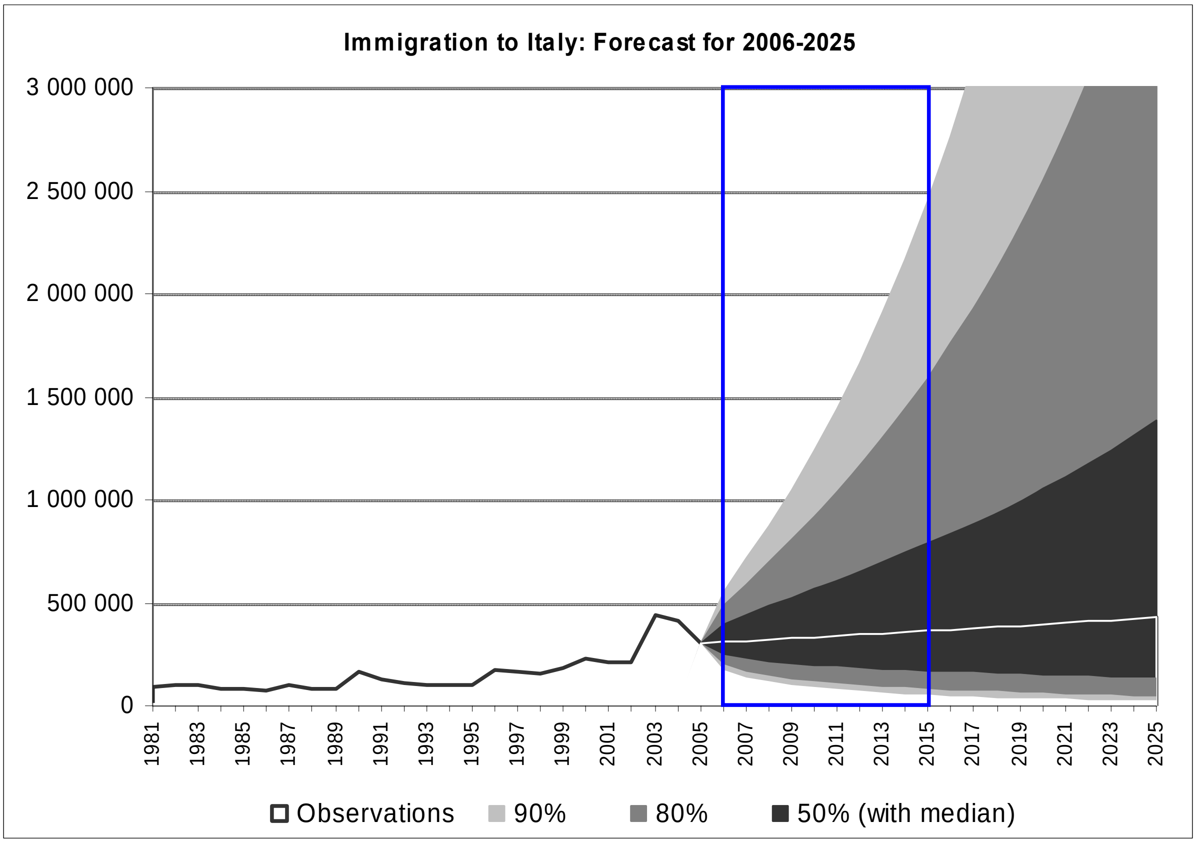


Notes: The last observation (2006) indicated 21 520 immigrants to Hungary. For scenarios (Eurostat/Europop 2008), see report. The figures do not include the predictive uncertainty of two additional demographic variables.

Source: Eurostat, Hungarian Central Statistical Office. Forecast: own computations.

B.7. Italy

Figure B5.1. Immigration to Italy, Random Walk model with constant variance, $p(M_2) = 0.994$



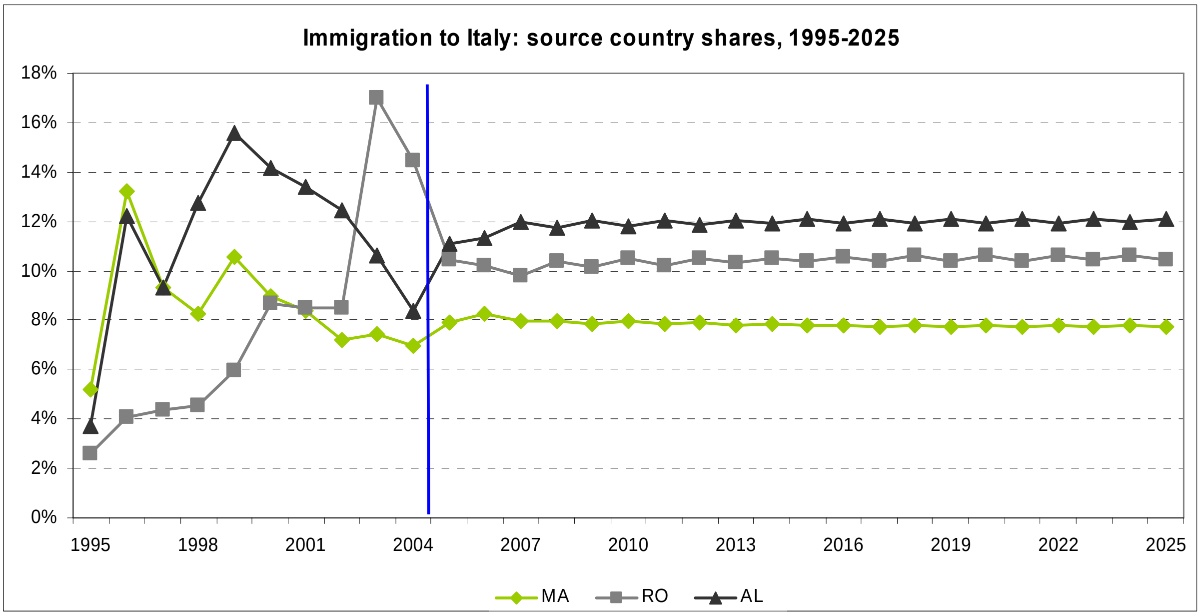
Note: the frame indicates a 10-year forecast horizon (2006–2015)
Source: Data until 2007: Eurostat and Italian National Institute of Statistics; forecast: own computations.

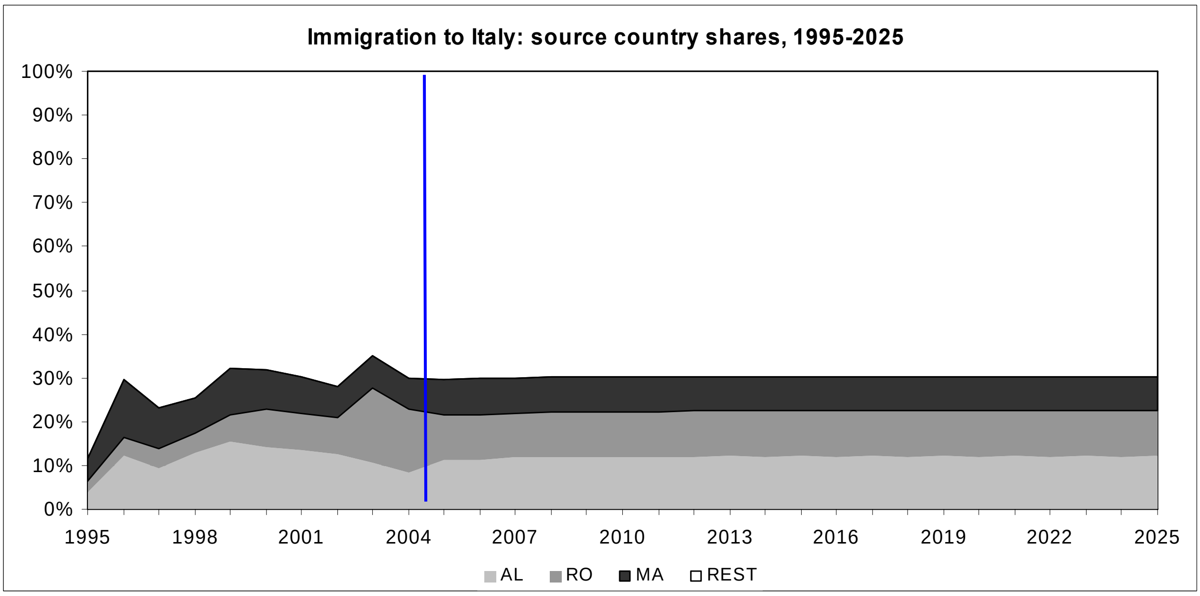
Table B5.1. Summary of predictive distributions for Italy: Median and 50-percent intervals (quartiles)

Year	1st Quartile	Median	3rd Quartile
2005 (*)	-	**304 960**	-
2006	247 707	**311 763**	392 385
2008	215 346	**321 258**	484 077
2010	194 853	**334 369**	568 070
2015	167 711	**365 858**	790 167
2020	150 242	**396 329**	1 056 001
2025	135 944	**433 653**	1 383 324

(*) 2007 – last observation in the sample
Source: As in Figure B5.1.

Figure B5.2. Immigration to Italy, source country shares, median forecasts (lower graph: cumulative)





Note: Data until 2003, forecast for 2004–2025. Codes: AL – Albania, RO – Romania, MA – Morocco
Source: As in Figure B5.1.

Table B5.2. Immigration to Italy, source country shares, median forecasts

Year	Total	MA	RO	AL	Rest
2004 (*)	414 880	31 009	64 290	37 195	282 386
2006	311 763	25 721	31 891	35 315	218 836
2008	321 258	25 638	33 284	37 791	224 545
2010	334 369	26 554	35 134	39 416	233 265
2015	365 858	28 449	38 020	44 173	255 216
2020	396 329	30 942	42 187	47 281	275 919
2025	433 653	33 517	45 289	52 355	302 492

(*) 2004 – last observation in the sample. Codes: AL – Albania, RO – Romania, MA – Morocco
Source: As in Figure B5.1.

Table B5.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	0.001	0	400	0	0.001	0	400
ϕ_i	Normal (*a*,*b*)	0.5	0.25	-	-	0.5	0.25	-	-
γ_i	Normal (*a*,*b*)	0.5	1	-	-	0.5	1	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.6371	2	0.6371	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B5.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$	$\begin{pmatrix}10&1&1\\1&10&1\\1&1&10\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B5.3.

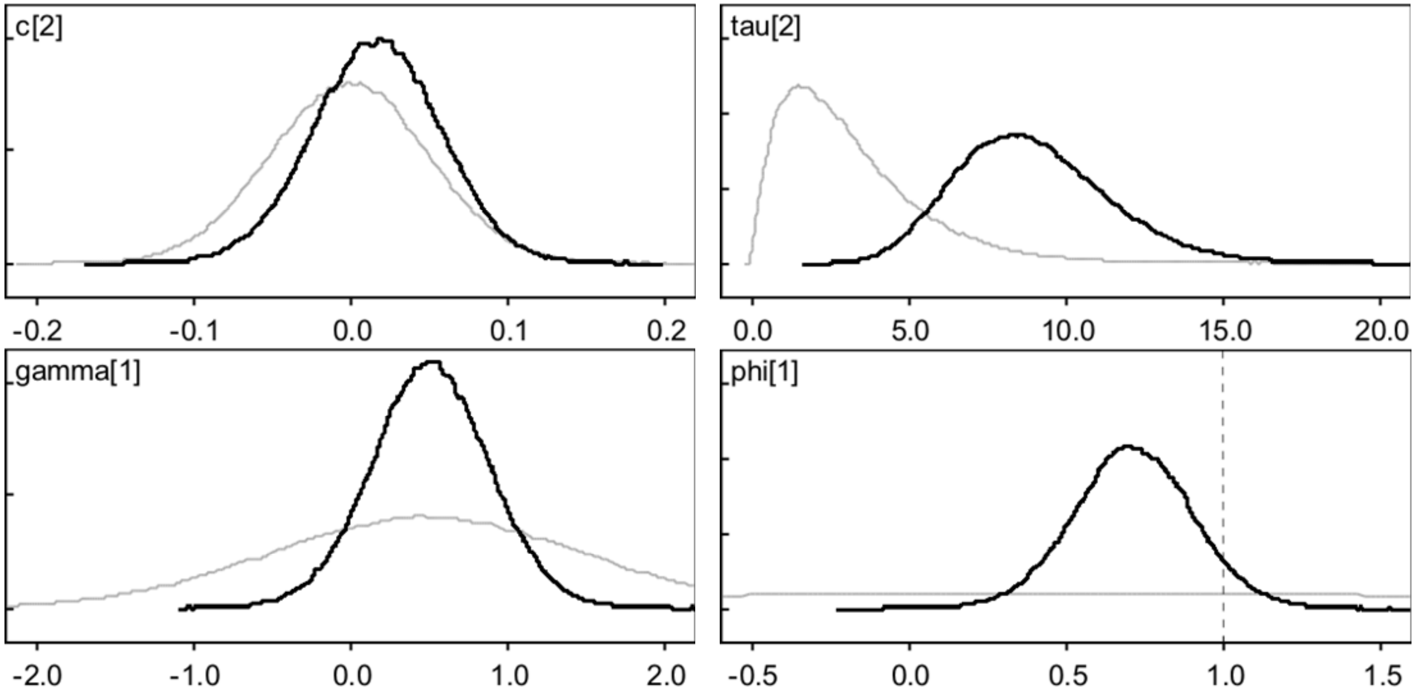
Table B5.5. Prior and posterior probabilities for various models (M_1 – M_4)

What did the experts expect? (*a priori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**17.4%**	**44.6%**
Random walk	**10.6%**	**27.4%**

How the data changed it? (*a posteriori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**0.1%**	**0.0%**
Random walk	**99.4%**	**0.5%**

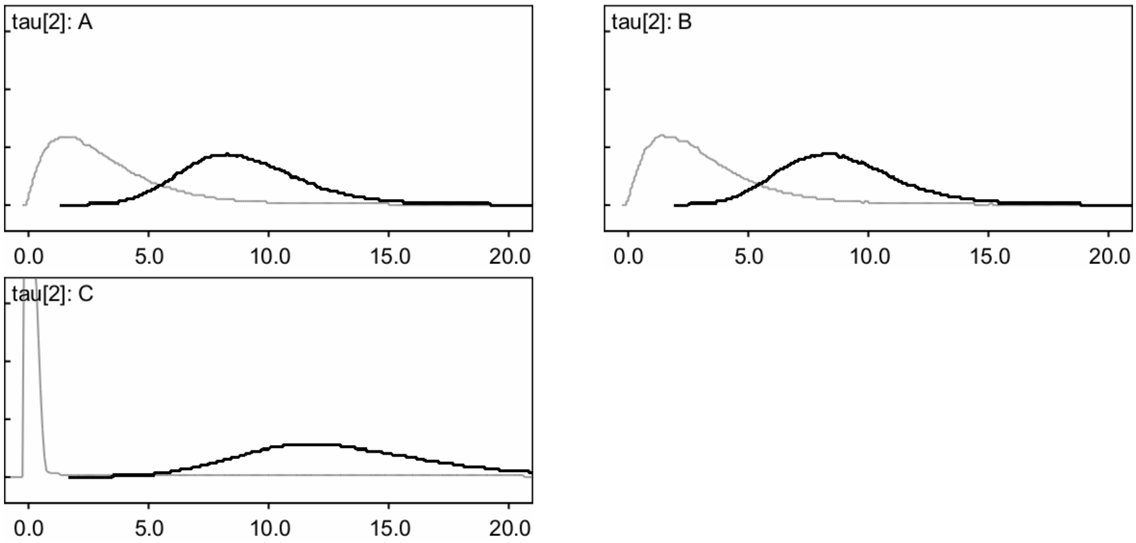
Source: Own elaboration in WinBUGS 3.0.3.

Figure B5.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

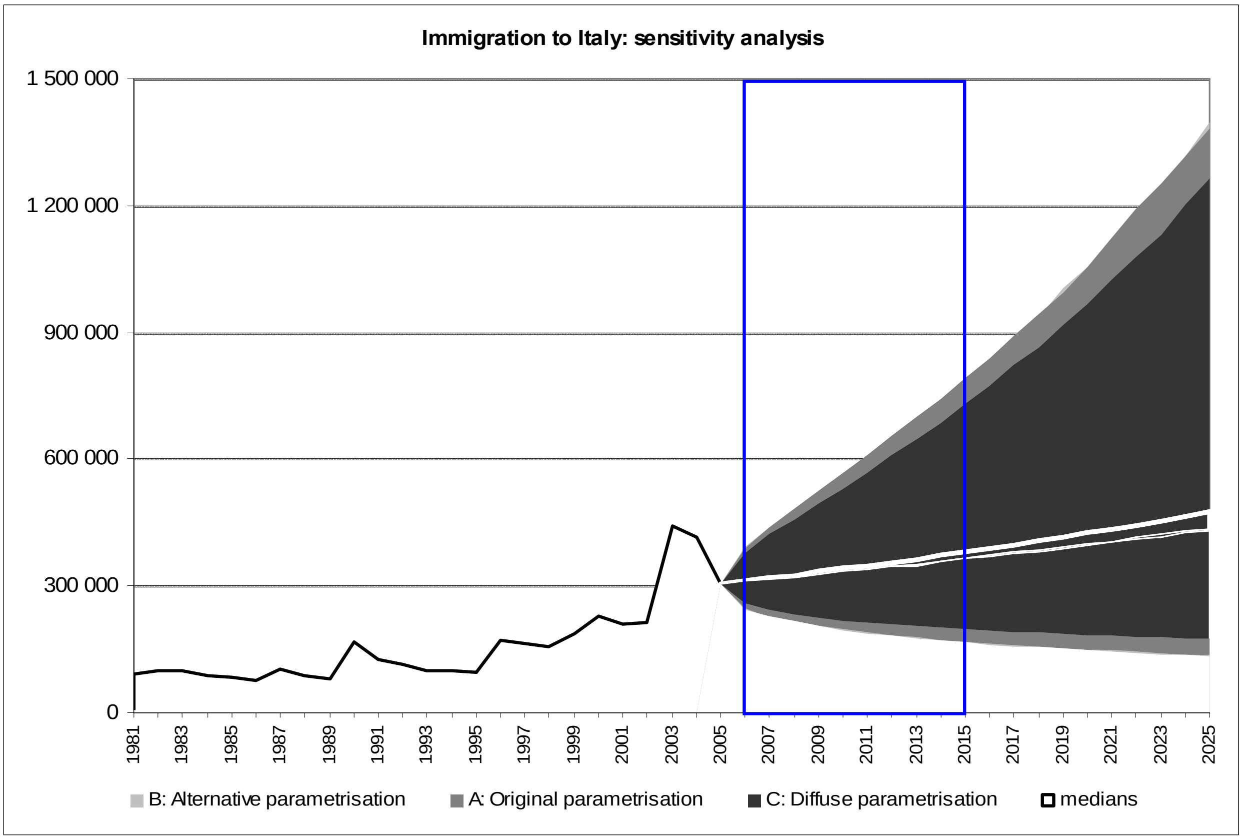
Figure B5.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.6371), B – alternative Γ(2.0972, 0.6681), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B5.5. Immigration to Italy, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.6371), B – Γ(2.0972, 0.6681), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2006–2015).

Source: Data until 2005: Eurostat and Italian National Institute of Statistics; forecast: own computations.

Table B5.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model		Demographic model	
		a	*b*	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.5 & 1.32 & -0.68\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}0.25 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.5 & -0.95 & -1.23\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}0.25 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}1.98 & 1.66 & -0.68\\1.66 & 3 & 0\\-0.68 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}1.64 & -0.91 & -1.11\\-0.91 & 3 & 0\\-1.11 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively

(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B5.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

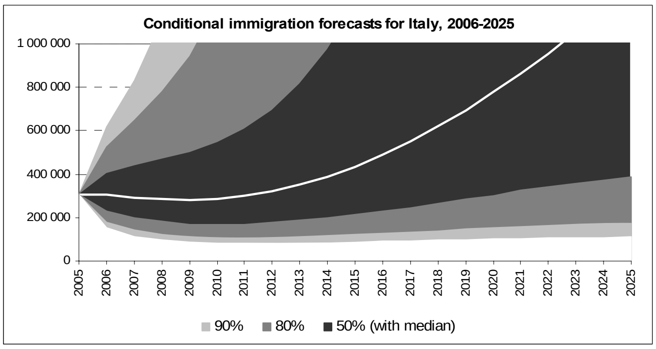
Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.226**	2.670	3.893	**4.069**	2.712	3.842
2nd (Lag)	**0.098**	2.692	3.940	**0.121**	0.187	0.269
Both (Lag)	**0.232**	4.664	6.278	**4.138**	2.789	3.915
1st (Inst)	**0.042**	2.689	3.928	**2.394**	2.687	3.878
2nd (Inst)	**1.080**	2.706	3.899	**4.477**	2.559	3.713
Both (Inst)	**1.117**	4.670	6.233	**7.044**	4.544	6.103

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

Table B5.8, Figure B5.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

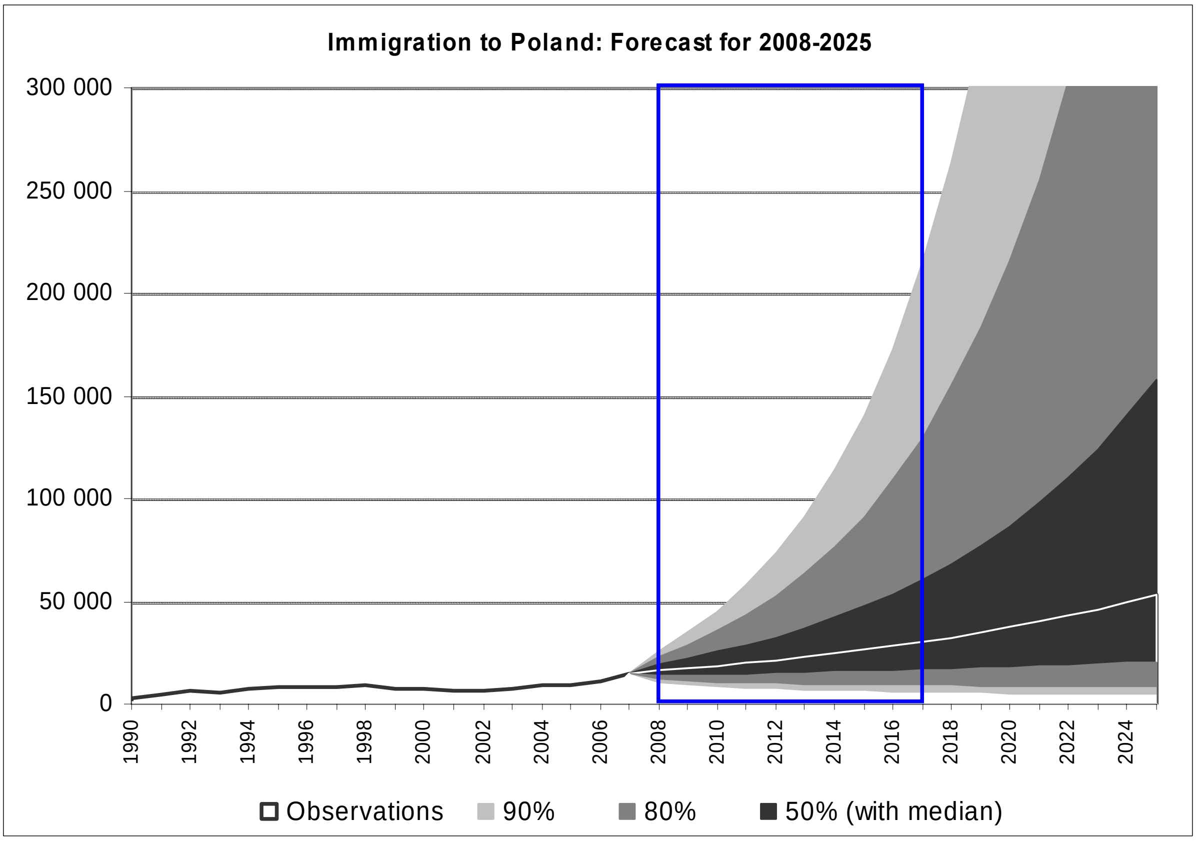
Year	Demographic model (general VAR)		
	1st Quartile	Median	3rd Quartile
2006	230 960	305 590	404 335
2008	181 680	284 930	469 771
2010	167 711	284 930	545 796
2015	213 203	433 653	1 178 791
2020	302 549	782 305	3 541 284
2025	388 481	1 251 683	Not plausible



Notes: The last observation (2005) indicated 304 960 immigrants to Italy. For scenarios (Eurostat/Europop 2008), see report. The figures do not include the predictive uncertainty of two additional demographic variables.
Source: Eurostat, Italian National Institute of Statistics. Forecast: own computations.

B.8. Poland

Figure B6.1. Immigration to Poland, averaged forecasts from Random Walk – Constant Variance and Random Walk – Stochastic Variance models, $p(M_2) = 0.469$ and $p(M_4) = 0.531$



Note: the frame indicates a 10-year forecast horizon (2008–2017)

Source: Data until 2007: Eurostat and Polish Central Statistical Office; forecast: own computations.

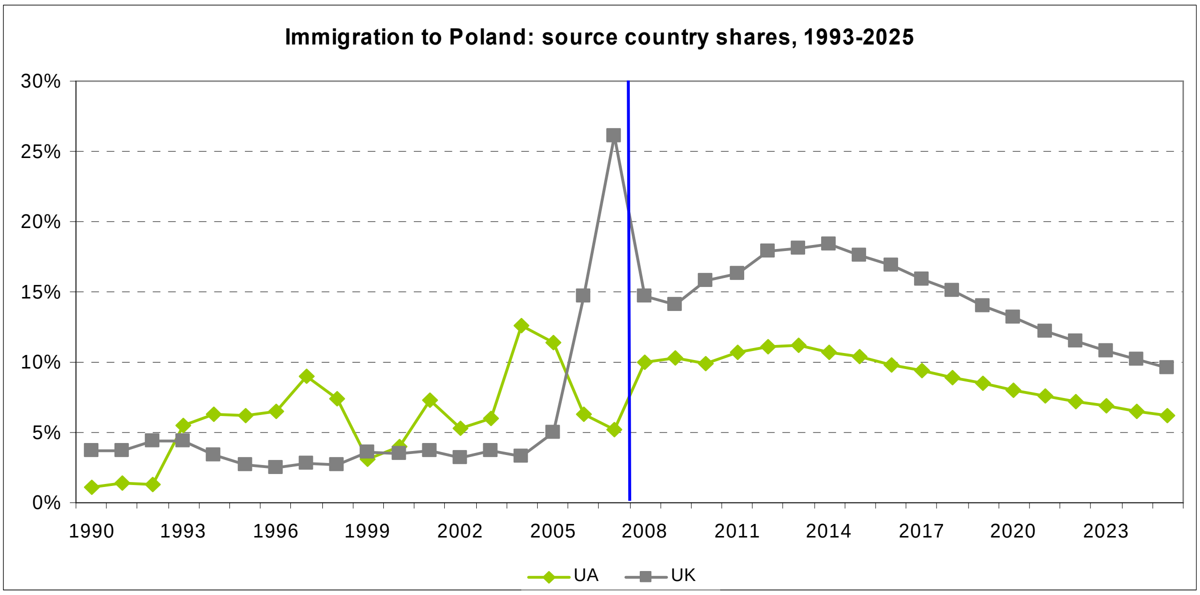
Table B6.1. Summary of predictive distributions for Poland: Median and 50-percent intervals (quartiles)

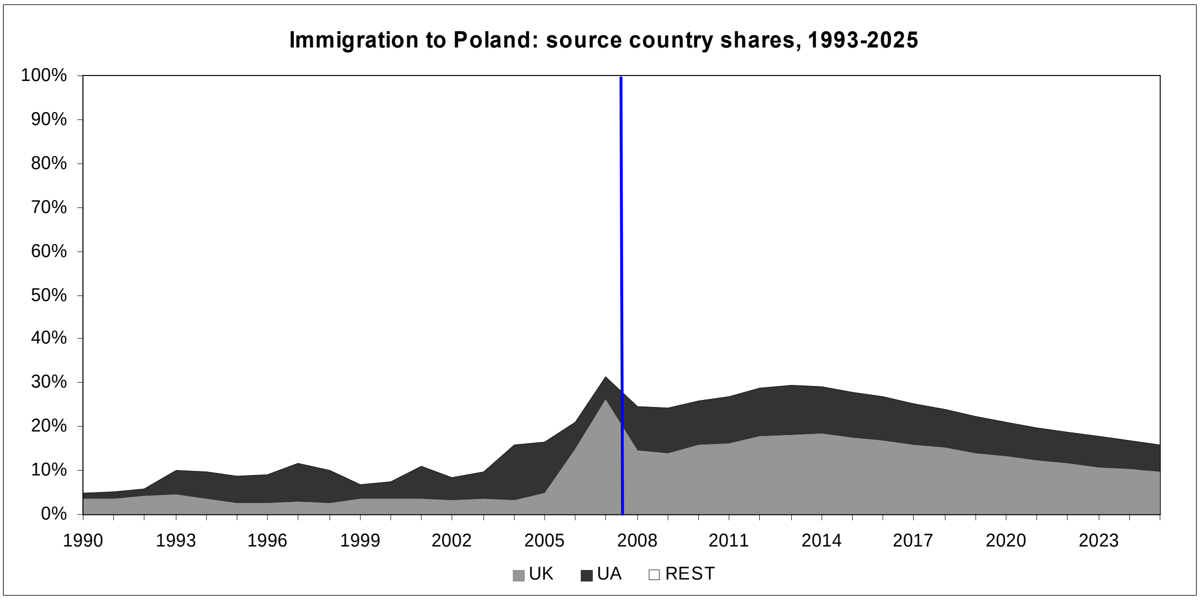
Year	1st Quartile	Median	3rd Quartile
2007 (*)	-	**14 995**	-
2008	13 849	**16 075**	18 883
2009	13 836	**17 257**	21 982
2010	13 989	**18 527**	25 336
2015	15 522	**26 370**	47 572
2020	17 730	**37 421**	86 682
2025	20 455	**53 104**	157 945

(*) 2007 – last observation in the sample

Source: As in Figure B6.1.

Figure B6.2. Immigration to Poland, source country shares, median forecasts (lower graph: cumulative)





Note: Data until 2007, forecast for 2008–2025. Codes: UA – Ukraine, UK – United Kingdom
Source: As in Figure B6.1.

Table B6.2. Immigration to Poland, source country shares, median forecasts

Year	Total	UA	UK	Rest
2007 (*)	14 995	777	3 913	10 305
2008	16 075	1 600	2 363	12 111
2009	17 257	1 785	2 429	13 044
2010	18 527	1 842	2 936	13 749
2015	26 370	2 737	4 641	18 992
2020	37 421	2 990	4 930	29 501
2025	53 104	3 301	5 094	44 708

(*) 2007 – last observation in the sample. Codes: UA – Ukraine, UK – United Kingdom
Source: As in Figure B6.1.

Table B6.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*B*
c_i	Normal (*a*,*b*)	11.4076	0.0122	0	0.0001	11.4076	0.0122	0	0.0001
ϕ_i	Normal (*a*,*b*)	0.5	4	-	-	0.5	4	-	-
γ_i	Beta (*a*,*b*)	20	2	-	-	20	2	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.4463	2	0.4463	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Normal (*a*,*b*)	-	-	-	-	0	1.0E-6	0	1.0E-6
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B6.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*B*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 1 & 49 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	2
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B6.3.

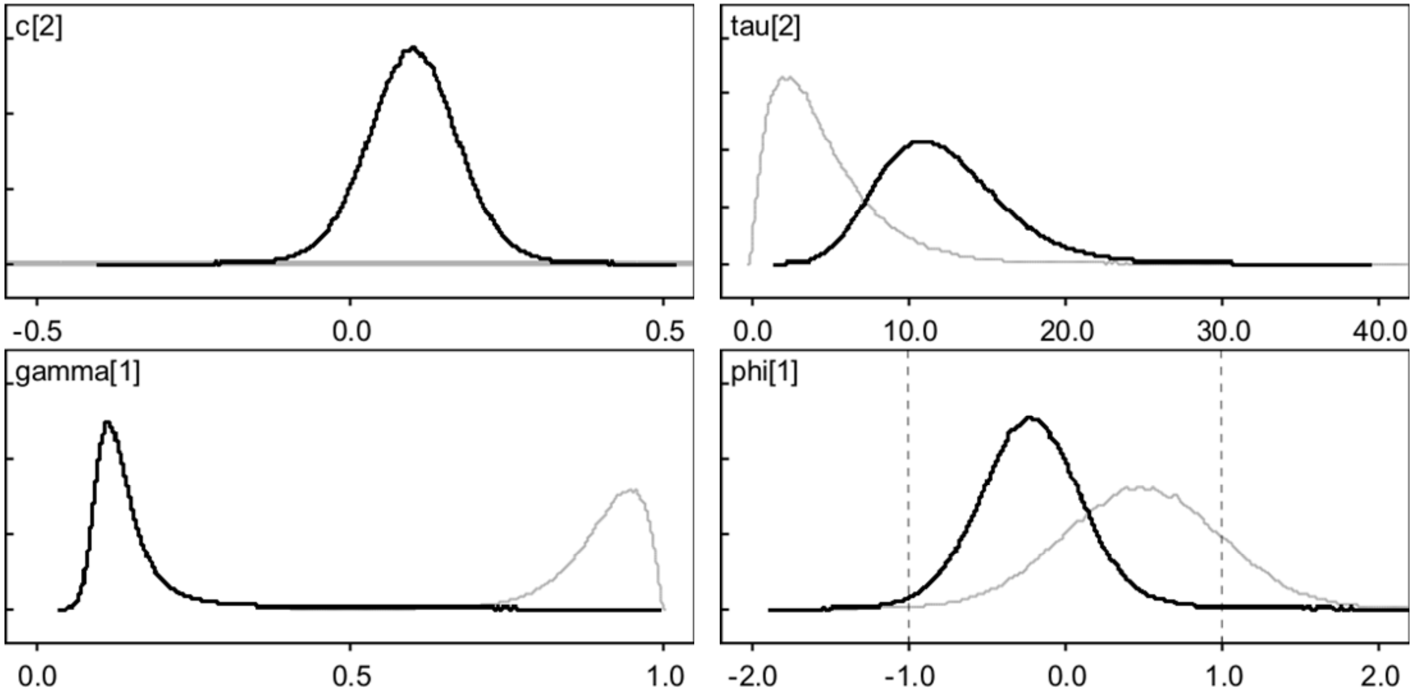
Table B6.5. Prior and posterior probabilities for various models (M_1 – M_4)

What did the experts expect? (*a priori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**29.7%**	**36.3%**
Random walk	**15.3%**	**18.7%**

How the data changed it? (*a posteriori*)		
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**0.0%**	**0.0%**
Random walk	**46.9%**	**53.1%**

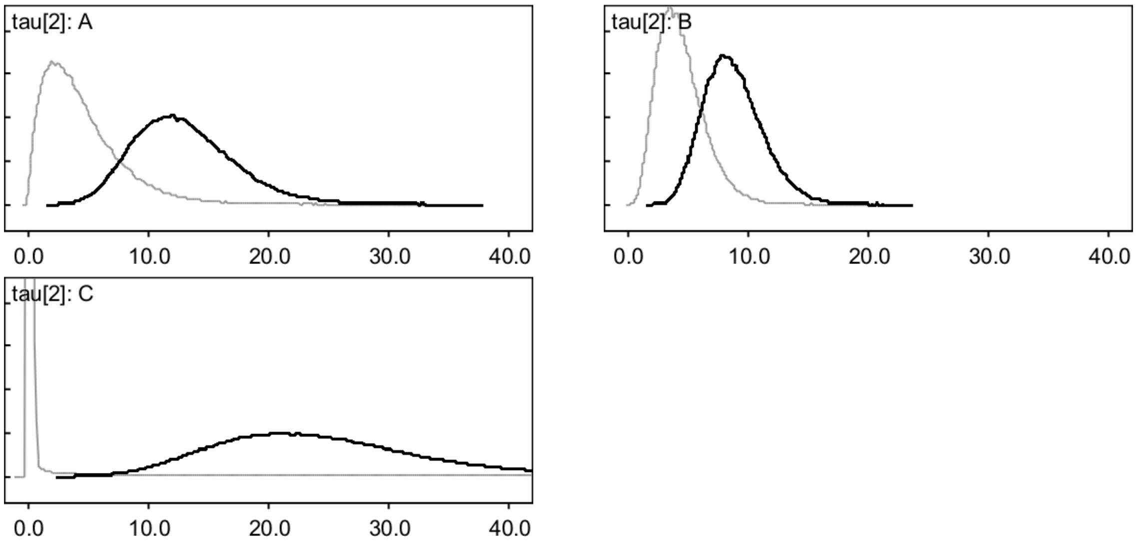
Source: Own elaboration in WinBUGS 3.0.3.

Figure B6.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

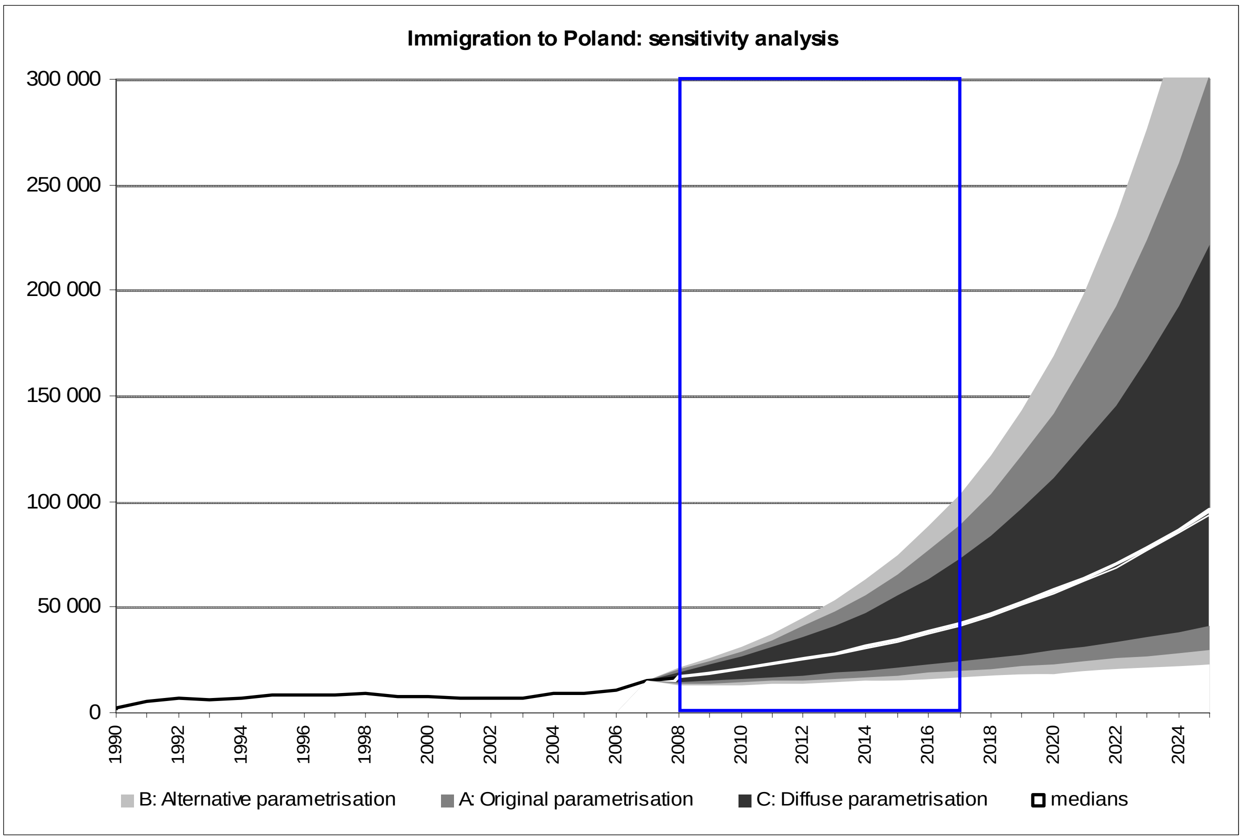
Figure B6.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.0463), B – alternative Γ(5.3336, 1.19), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B6.5. Immigration to Poland, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.0463), B – alternative Γ(5.3336, 1.19), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2008–2017).

Source: Data until 2007: Eurostat and Polish Central Statistical Office; forecast: own computations.

Table B6.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model		Demographic model	
		a	*b*	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$
A = [α_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.5 & 1.07 & -0.89\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}4 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.5 & -0.29 & -1.25\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}4 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}2.93 & 2.33 & -1.17\\2.33 & 3 & 0\\-1.17 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}1.10 & -0.26 & -1.10\\-0.26 & 3 & 0\\-1.10 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B6.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

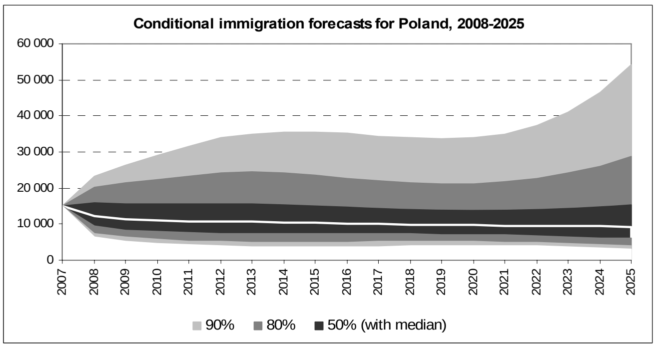
Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**0.177**	2.655	3.924	**0.114**	2.711	3.841
2nd (Lag)	**0.027**	2.663	3.932	**0.075**	2.717	3.975
Both (Lag)	**0.197**	4.658	6.315	**0.357**	4.660	6.157
1st (Inst)	**0.191**	2.679	3.918	**0.324**	2.685	3.926
2nd (Inst)	**0.374**	2.653	3.933	**6.551**	2.676	3.941
Both (Inst)	**0.872**	4.650	6.323	**6.901**	4.687	6.337

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

Table B6.8, Figure B6.6. Summaries of conditional (scenario-based) forecasts from the demographic VAR models

Year	Demographic model (general VAR)		
	1st Quartile	Median	3rd Quartile
2008	9 490	12 308	15 867
2009	8 392	11 419	15 646
2010	7 871	11 026	15 631
2015	7 237	10 394	15 063
2020	7 066	9 691	13 671
2025	6 039	9 265	15 199

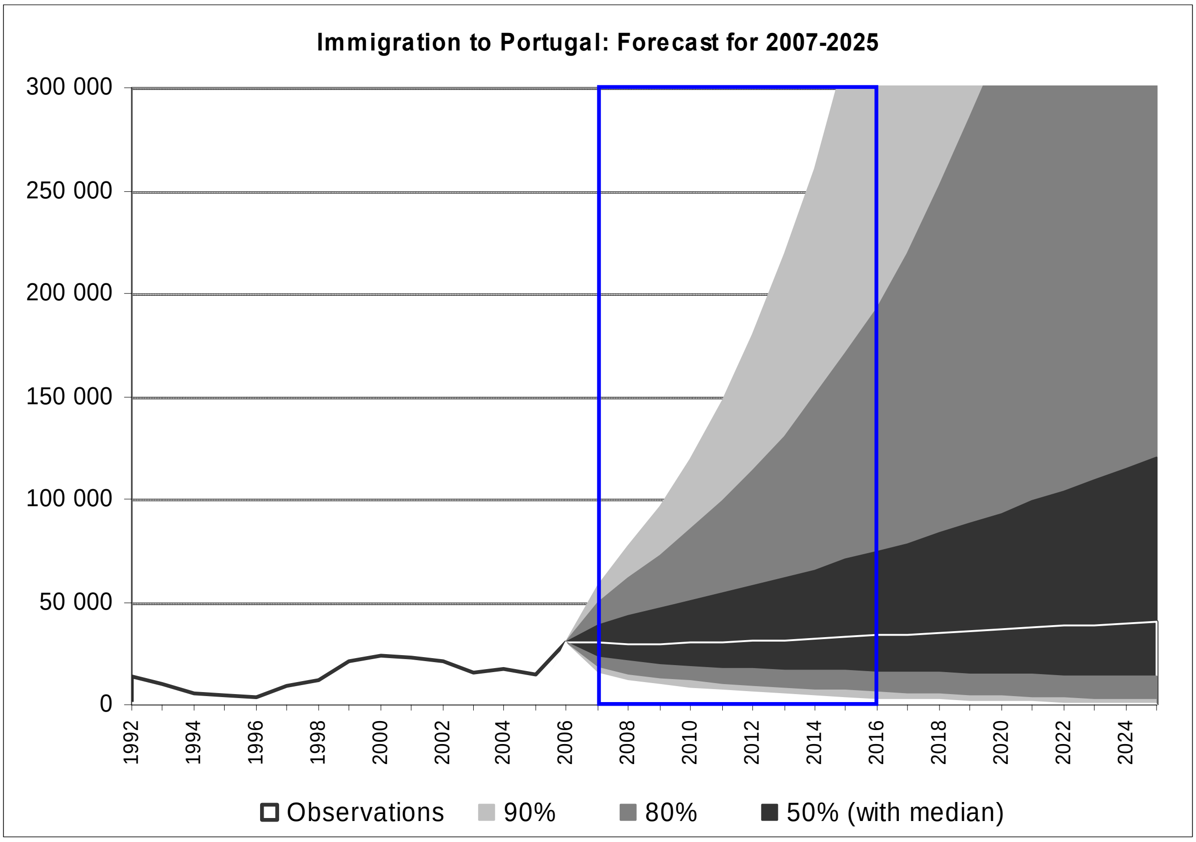


Notes: The last observation (2007) indicated 14 995 immigrants to Poland. For scenarios (Eurostat/Europop 2008), see report. The figures do not include the predictive uncertainty of two additional demographic variables.

Source: Eurostat, Polish Central Statistical Office. Forecast: own computations.

B.9. Portugal

Figure B7.1. Immigration to Portugal, averaged forecasts from Random Walk and Autoregression models with constant variance, $p(M_2) = 0.633$ and $p(M_1) = 0.367$



Note: the frame indicates a 10-year forecast horizon (2007–2016)

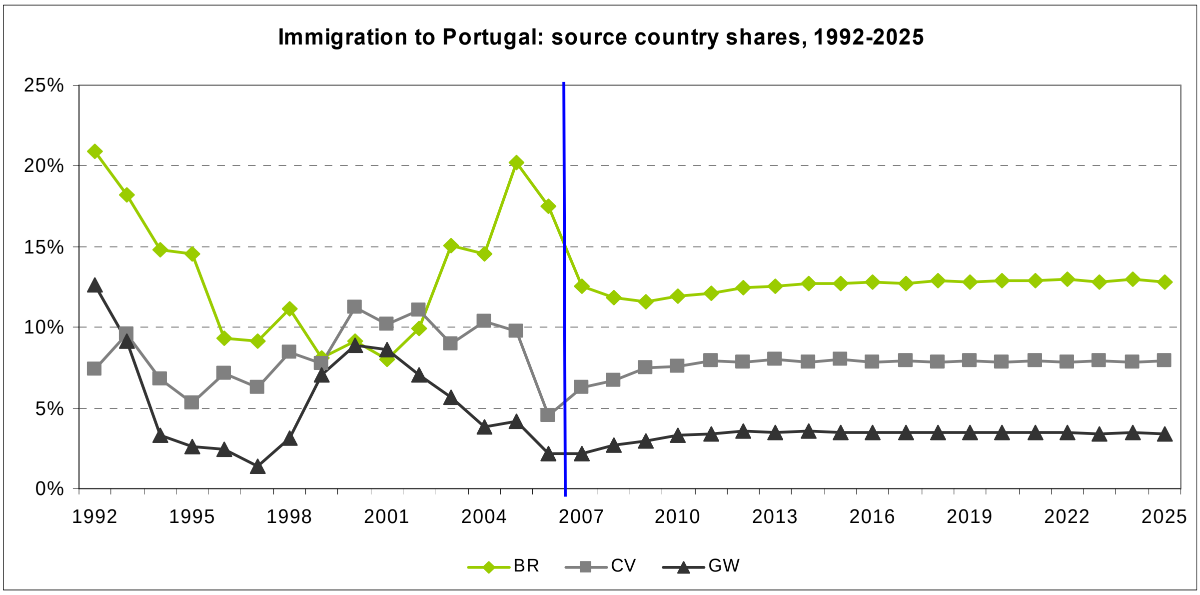
Source: Data until 2006: Eurostat and Statistics Portugal with own recalculations; forecast: own computations.

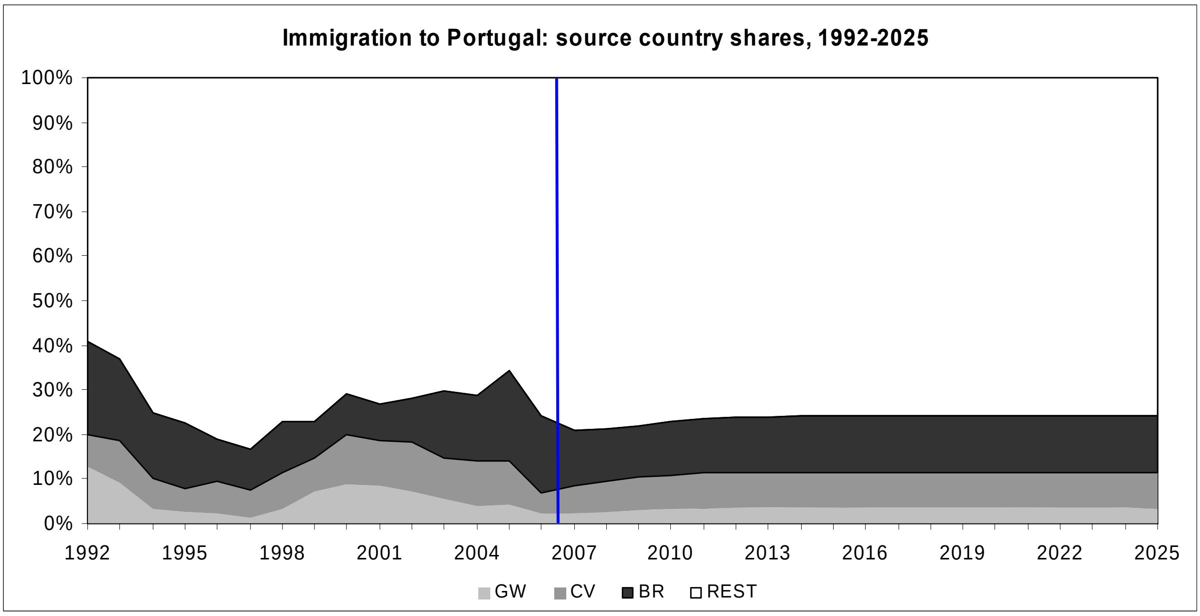
Table B7.1. Summary of predictive distributions for Portugal: Median and 50-percent intervals (quartiles)

Year	1st Quartile	Median	3rd Quartile
2006 (*)	-	**30 727**	-
2007	23 156	**30 031**	38 949
2008	20 889	**29 733**	43 045
2010	18 583	**30 031**	50 514
2015	16 220	**33 190**	70 263
2020	14 750	**36 680**	92 967
2025	13 413	**40 135**	120 572

(*) 2006 – last observation in the sample
Source: As in Figure B7.1.

Figure B7.2. Immigration to Portugal, source country shares, median forecasts (lower graph: cumulative)





Note: Data until 2006, forecast for 2007–2025. Codes: BR – Brazil, CV – Cape Verde, GW – Guinea Bissau
Source: As in Figure B7.1.

Table B7.2. Immigration to Portugal, source country shares, median forecasts

Year	Total	BR	CV	GW	Rest
2006 (*)	30 727	5 384	1 402	662	23 278
2007	30 031	3 768	1 873	667	23 724
2008	29 733	3 515	1 990	798	23 430
2010	30 031	3 578	2 274	990	23 190
2015	33 190	4 212	2 646	1 155	25 177
2020	36 680	4 741	2 870	1 288	27 781
2025	40 135	5 145	3 171	1 373	30 446

(*) 2006 – last observation in the sample. Codes: BR – Brazil, CV – Cape Verde, GW – Guinea Bissau
Source: As in Figure B7.1.

Table B7.3. Prior distributions characteristics of the models for total immigration

Model M_i		M_1: AR(1)-CV		M_2: RW-CV		M_3: AR(1)-SV		M_4: RW-SV	
Parameter	Distribution	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
c_i	Normal (*a*,*b*)	0	0.001	0	0.1	0	0.001	0	0.1
ϕ_i	Normal (*a*,*b*)	0.4	1.5625	-	-	0.4	1.5625	-	-
γ_i	Normal (*a*,*b*)	0.5	1	-	-	0.5	1	-	-
$\tau_i = 1/\sigma_i^2$	Gamma (*a*,*b*)	2	0.1345	2	0.1345	-	-	-	-
K_i	Normal (*a*,*b*)	-	-	-	-	0	1	0	1
ψ_i	Uniform (*a*,*b*)	-	-	-	-	–0.99	0.99	–0.99	0.99
$\rho_i = 1/\upsilon_i^2$	Gamma (*a*,*b*)	-	-	-	-	1	1	1	1
b_i (dummy)	Normal (a,b)	0	0.0001	0	0.0001	0	0.0001	0	0.0001

Models: AR(1) – autoregressive model, RW – random walk model, CV – constant variance, SV – stochastic variance
Source: Own elaboration on the basis of the Delphi expert survey.

Table B7.4. Prior distributions characteristics of the VAR model for source countries

Parameter	Distribution	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	3
b = [b_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0.001 \\ 0.001 \\ 0.001 \end{pmatrix}$

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively
(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom
Source: As in Table B7.3.

Table B7.5. Prior and posterior probabilities for various models ($M_1 - M_4$)

What did the experts expect? (*a priori*)

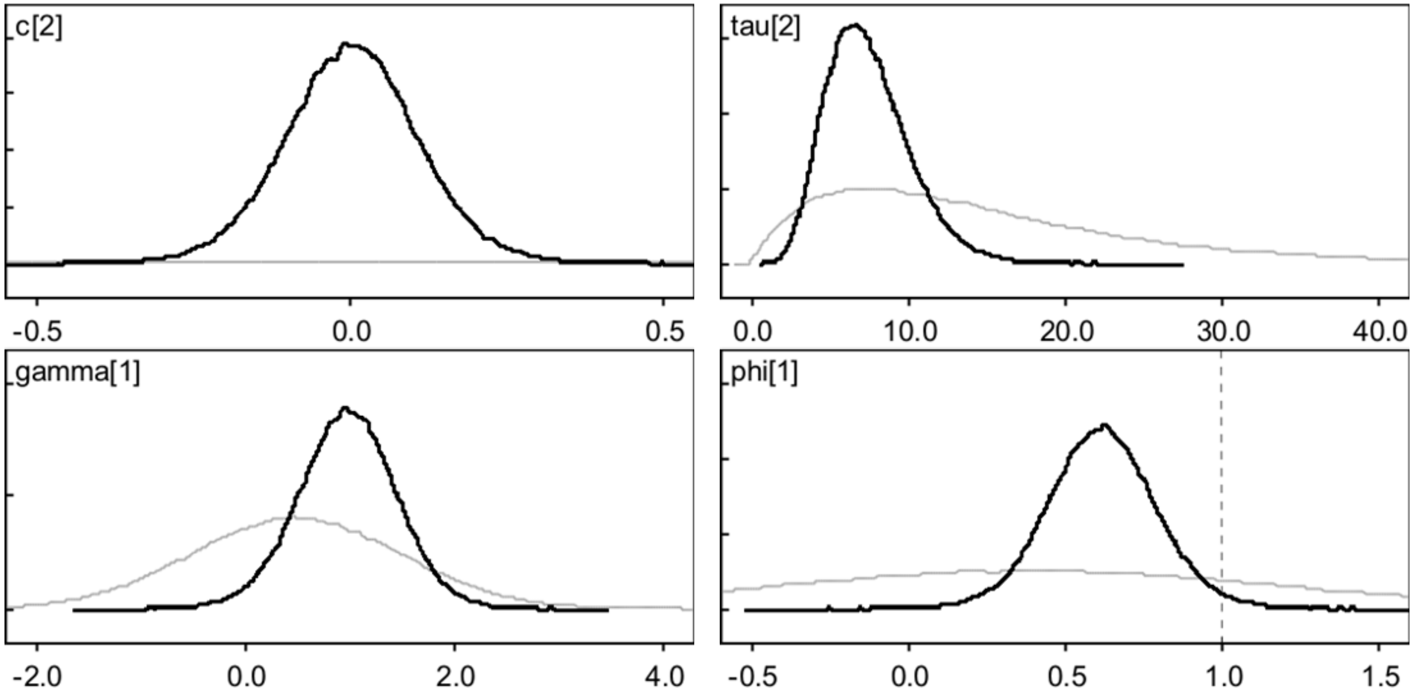
Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**38.4%**	**41.6%**
Random walk	**9.6%**	**10.4%**

How the data changed it? (*a posteriori*)

Probabilities:	Constant variance	Random variance
Autoregressive process with trend	**36.7%**	**0.0%**
Random walk	**63.3%**	**0.0%**

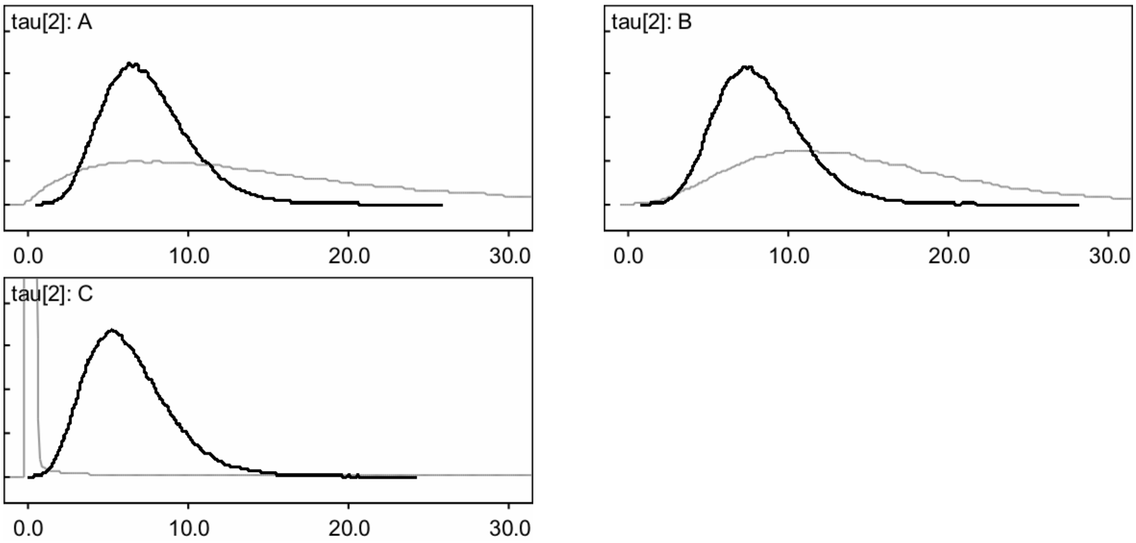
Source: Own elaboration in WinBUGS 3.0.3.

Figure B7.3. Examples of prior (grey) and posterior (black) distributions of the selected model parameters (common vertical scales): c_2, τ_2, γ_1 and ϕ_1



Note: numbers in square brackets indicate particular models
Source: Own elaboration in WinBUGS 3.0.3.

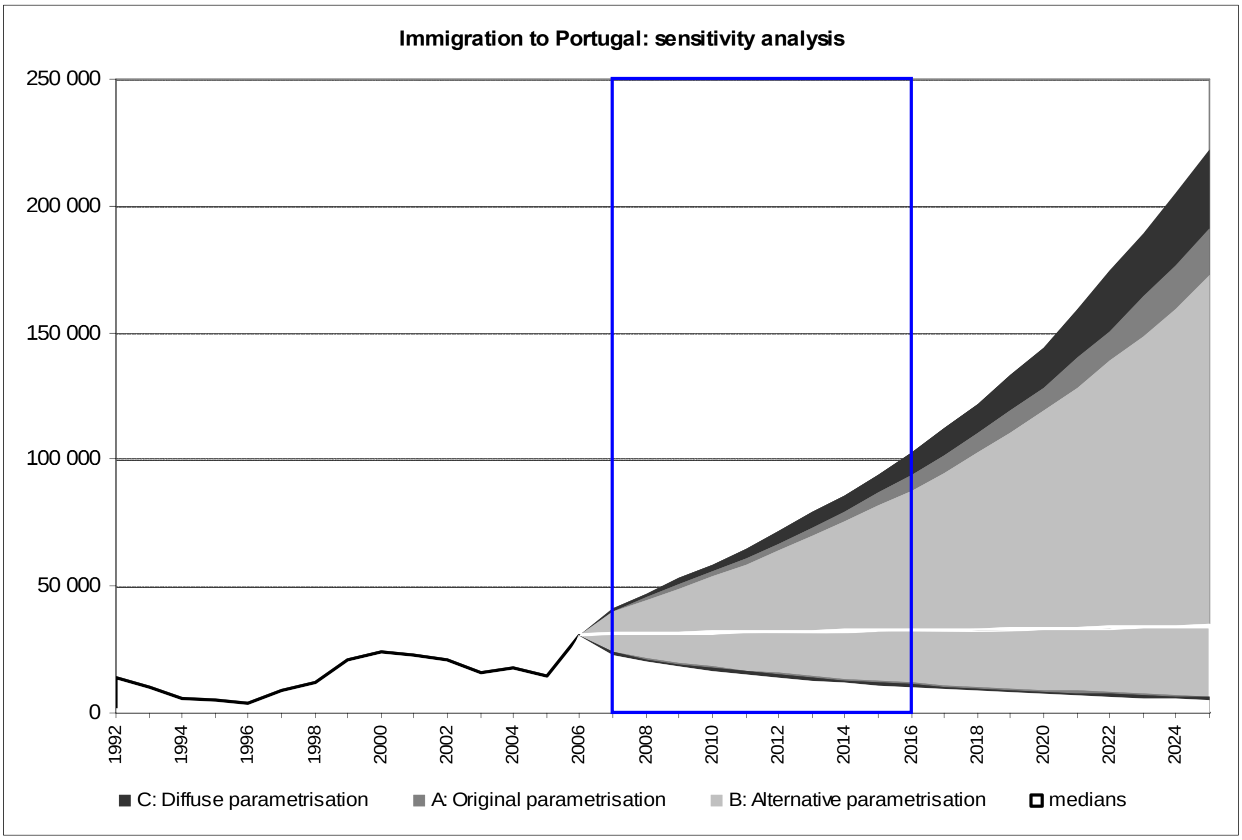
Figure B7.4. Example of the sensitivity analysis: various prior (grey) and posterior (black) distributions for τ_2



Note: The priors are: A – the originally used Γ(2, 0.1345), B – alternative Γ(4.0111, 0.2698), C – diffuse Γ(0.01, 0.01). All figures are shown in comparable scales.

Source: Own elaboration in WinBUGS 3.0.3.

Figure B7.5. Immigration to Portugal, Random Walk models with various prior distributions for τ_2



Notes: The graph depicts median values and 50-percent predictive intervals of forecasts obtained under various priors for τ_2. These priors are: A – the originally used Γ(2, 0.1345), B – alternative Γ(4.0111, 0.2698), and C – diffuse Γ(0.01, 0.01). The frame indicates a 10-year forecast horizon (2007–2016).

Source: Data until 2006: Eurostat and Statistics Portugal; forecast: own computations.

Table B7.6. Prior distributions characteristics of the VAR models for additional demo-economic variables

Parameter	Distribution	Economic model		Demographic model	
		a	*b*	*a*	*b*
c = [c_i]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$	$\begin{pmatrix}0\\0\\0\end{pmatrix}$	$\begin{pmatrix}0.0001\\0.0001\\0.0001\end{pmatrix}$
ϕ = [ϕ_{ij}]	Normal (*a*,*b*) (*)	$\begin{pmatrix}0.4 & 1.39 & -0.67\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.56 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$	$\begin{pmatrix}0.4 & -0.39 & -0.94\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$	$\begin{pmatrix}1.56 & 4 & 4\\1 & 0.01 & 1\\1 & 1 & 0.01\end{pmatrix}$
T = [τ_{ij}]	Wishart (*a*,*b*) (**)	$\begin{pmatrix}0.39 & 0.66 & -0.36\\0.66 & 3 & 0\\-0.36 & 0 & 3\end{pmatrix}$	3	$\begin{pmatrix}0.42 & -0.44 & -0.69\\-0.44 & 3 & 0\\-0.69 & 0 & 3\end{pmatrix}$	3

(*) In case of Normal distributions, *a* and *b* are a vector or a matrix of expected values and precisions, respectively

(**) For the Wishart distribution, the parameter *b* denotes degrees of freedom

Source: Own elaboration, partially based on the Delphi expert survey.

Table B7.7. Results of Lindley-type Wald's tests for the impact of demo-economic variables on immigration

Variables tested *	Economic model			Demographic model		
	Test statistic	90% quantile	95% quantile	Test statistic	90% quantile	95% quantile
1st (Lag)	**3.661**	2.621	3.956	**0.679**	2.706	3.839
2nd (Lag)	**0.034**	2.638	3.903	**0.227**	2.653	3.892
Both (Lag)	**3.903**	4.645	6.419	**0.944**	4.612	6.045
1st (Inst)	**0.175**	2.654	3.957	**0.484**	2.659	3.975
2nd (Inst)	**1.378**	2.637	3.973	**1.283**	2.652	3.968
Both (Inst)	**2.655**	4.691	6.498	**1.802**	4.706	6.551

* The '**1st**' variable denotes GDP growth (economic model) or natural population growth (demographic model), and
the '**2nd**' – respectively unemployment rates or shares of population aged 15–64. Tests are done for the **Lag**[ged] and **Inst**[antaneous] impact, which is found significant if the test statistic is higher than the (1–significance level) quantile.

Source: Own elaboration in WinBUGS 3.0.3.

■

Appendix C Sample WinBUGS code for forecasting tasks

B.10. Models estimation and forecasts: example of the Czech Republic

```
# Models estimation and forecasts

model
{ # Priors for estimation - INFORMATIVE
    c[1] ~ dnorm(0,0.001); c[3] ~ dnorm(0,0.001)  # Uncertain policy constants (answers (a))
    c[2] ~ dnorm(0,400); c[4] ~ dnorm(0,400)  # Random-walk constants, concentrated in 0

    b[2] ~ dnorm(-1,1); b[4] ~ dnorm(-1,1) # Adjustment for definition change in random walks

    mu.tau <- 2*log((0.4385)*(5/4)*(0.4385)*(5/4)+1)
    for (i in 2:n) { for (k in 1:2) { tau[i,k] <- theta[k] } }
    for (k in 1:2) { theta[k] ~ dgamma(2,mu.tau) } # Informative - X~logN(.,.), logX~N(.,.)

    phi[1] ~ dnorm(0.396,1.778); phi[3] ~ dnorm(0.396,1.778) # 0.58/0.71 vs 0.13/0.71
       gam[1] ~ dnorm(0,0.0001); gam[3] ~ dnorm(0,0.0001)  # Hardly informative

       #SV part
       for (k in 3:4) { K[k] ~ dnorm(0,1) }
       for (k in 3:4) { psi[k] ~ dunif(-0.99,0.99) }
       for (k in 3:4) { rho[k] ~ dgamma(1,1) }

# Data transformation
for (i in 1:n) {for (k in 1:4) { cz[i,k] <- log(CZ[i]) } }

# Initial tau for SV
       tau[1,3] ~ dgamma(1,1); tau[1,4] ~ dgamma(1,1)

# Model
       for (i in 2:n) {
              mu[i,1] <- c[1]+gam[1]*log(log(tt[i]))+phi[1]*cz[i-1,1]
              mu[i,2] <- c[2]+b[2]*dummy[i]+cz[i-1,2]
              mu[i,3] <- c[3]+gam[3]*log(log(tt[i]))+phi[3]*cz[i-1,3]
              mu[i,4] <- c[4]+b[4]*dummy[i]+cz[i-1,4]

              #SV auxiliary
              for (k in 3:4) {
                     lv[i,k]<- K[k]+psi[k]*log(1/tau[i-1,k])
                     tmp[i,k] <- lv[i,k]+z[i,k]
                     tau[i,k] <- exp(-tmp[i,k])
                     z[i,k] ~ dnorm(0,rho[k]) }

       #  Model update
       for (k in 1:4) { cz[i,k] ~ dnorm(mu[i,k], tau[i,k])  }
       }

       # Forecasting
       # Last observation - anchor for the forecasts
       for (k in 1:4) { cz.new[n,k] <- cz[n,k] }
       tau.new[n,3] <- tau[n,3]; tau.new[n,4] <- tau[n,4]

       for (i in n+1:n+N) {
              mu.new[i,1] <- c[1]+gam[1]*log(log(tt[i]))+phi[1]*cz.new[i-1,1]
              mu.new[i,2] <- c[2]+b[2]*dummy[i]+cz.new[i-1,2]
              mu.new[i,3] <- c[3]+gam[3]*log(log(tt[i]))+phi[3]*cz.new[i-1,3]
              mu.new[i,4] <- c[4]+b[4]*dummy[i]+cz.new[i-1,4]

              # CV auxiliary
              for (k in 1:2) { tau.new[i,k] <- theta[k] }

              # SV auxiliary
              for (k in 3:4) {
                     lv[i,k]<- K[k]+psi[k]*log(1/tau.new[i-1,k])
                     tmp[i,k] <- lv[i,k]+z[i,k]
                     tau.new[i,k] <- exp(-tmp[i,k])
                     z[i,k] ~ dnorm(0,rho[k]) }

     for (k in 1:4) { cz.new[i,k] ~ dnorm(mu.new[i,k], tau.new[i,k])  }

     }
```

}

B.11. Carlin-Chib model selection: example of the Czech Republic

```
# Model selection

model
{
# Priors on the model space
mod ~ dcat(p[]) # Categorical distribution
p[1]<-0.433; p[2]<-0.217; p[3]<-0.233; p[4]<-0.117;

# Carlin-Chib Priors
for (k in 1:4) { c[k] ~ dnorm(mu.c[k,mod],tau.c[k,mod]) }
b[2] ~ dnorm(mu.b[2,mod],tau.b[2,mod]); b[4] ~ dnorm(mu.b[4,mod],tau.b[4,mod])

# AR part
phi[1] ~ dnorm(mu.phi[1,mod],tau.phi[1,mod]); phi[3] ~ dnorm(mu.phi[3,mod],tau.phi[3,mod])
gam[1] ~ dnorm(mu.gam[1,mod],tau.gam[1,mod]); gam[3] ~ dnorm(mu.gam[3,mod],tau.gam[3,mod])

for (i in 2:n) { for (k in 1:2) { tau[i,k] <- theta[k] } }
for (k in 1:2) { theta[k] ~ dgamma(r.tau[k,mod],m.tau[k,mod]) }

# SV part
for (k in 3:4) { K[k] ~ dnorm(mu.K[k,mod],tau.K[k,mod])
                 psi[k] ~ dnorm(mu.psi[k,mod],tau.psi[k,mod])C(-0.99,0.99)
                 rho[k] ~ dgamma(r.rho[k,mod],m.rho[k,mod]) }

# Values of hyperparameters - for mod = k
mu.c[1,1] <- 0; tau.c[1,1] <- 0.001; mu.c[3,3] <- 0; tau.c[3,3] <- 0.001
mu.c[2,2] <- 0; tau.c[2,2] <- 400; mu.c[4,4] <- 0; tau.c[4,4] <- 400

mu.b[2,2] <- -1; tau.b[2,2] <- 1; mu.b[4,4] <- -1; tau.b[4,4] <- 1

mu.phi[1,1] <- 0.396; tau.phi[1,1] <- 1.778; mu.phi[3,3] <- 0.396; tau.phi[3,3] <- 1.778
mu.gam[1,1] <- 0; tau.gam[1,1] <- 0.0001; mu.gam[3,3] <- 0; tau.gam[3,3] <- 0.0001

mu.K[3,3] <- 0; tau.K[3,3] <- 1; mu.K[4,4] <- 0; tau.K[4,4] <- 1
mu.psi[3,3] <- 0; tau.psi[3,3] <- 0.000001; mu.psi[4,4] <- 0; tau.psi[4,4] <- 0.000001
r.rho[3,3] <- 1; m.rho[3,3] <- 1;r.rho[4,4] <- 1; m.rho[4,4] <- 1

for (k in 1:2) { r.tau[k,k] <- 2; m.tau[k,k] <- 2*log((0.4385)*(5/4)*(0.4385)*(5/4)+1)  }

# Values of hyperparameters - to be got in Phase 2 - for mod ≠ k
       mu.c[1,2] <- 5.3850; mu.c[1,3] <- 5.3850; mu.c[1,4] <- 5.3850;
       mu.c[2,1] <- 0.0275; mu.c[2,3] <- 0.0275; mu.c[2,4] <- 0.0275;
       mu.c[3,1] <- 6.0440; mu.c[3,2] <- 6.0440; mu.c[3,4] <- 6.0440;
       mu.c[4,1] <- 0.0321; mu.c[4,2] <- 0.0321; mu.c[4,3] <- 0.0321;

       tau.c[1,2] <- 0.0643; tau.c[1,3] <- 0.0643; tau.c[1,4] <- 0.0643;
       tau.c[2,1] <- 420.26; tau.c[2,3] <- 420.26; tau.c[2,4] <- 420.26;
       tau.c[3,1] <- 0.1041; tau.c[3,2] <- 0.1041; tau.c[3,4] <- 0.1041;
       tau.c[4,1] <- 420.26; tau.c[4,2] <- 420.26; tau.c[4,3] <- 420.26;

       mu.b[2,1] <- -0.0530; mu.b[4,1] <- -0.1691;
       mu.b[2,3] <- -0.0530; mu.b[4,2] <- -0.1691;
       mu.b[2,4] <- -0.0530; mu.b[4,3] <- -0.1691;

       tau.b[2,1] <- 34.971; tau.b[4,1] <- 63.592;
       tau.b[2,3] <- 34.971; tau.b[4,2] <- 63.592;
       tau.b[2,4] <- 34.971; tau.b[4,3] <- 63.592;

       mu.gam[1,2] <- 1.770; mu.gam[3,1] <- 2.071;
       mu.gam[1,3] <- 1.770; mu.gam[3,2] <- 2.071;
       mu.gam[1,4] <- 1.770; mu.gam[3,4] <- 2.071;

       tau.gam[1,2] <- 0.620; tau.gam[3,1] <- 0.7535;
       tau.gam[1,3] <- 0.620; tau.gam[3,2] <- 0.7535;
       tau.gam[1,4] <- 0.620; tau.gam[3,4] <- 0.7535;

       mu.phi[1,2] <- 0.4333; mu.phi[3,1] <- 0.3501;
       mu.phi[1,3] <- 0.4333; mu.phi[3,2] <- 0.3501;
       mu.phi[1,4] <- 0.4333; mu.phi[3,4] <- 0.3501;

       tau.phi[1,2] <- 5.5783; tau.phi[3,1] <- 5.9228;
       tau.phi[1,3] <- 5.5783; tau.phi[3,2] <- 5.9228;
       tau.phi[1,4] <- 5.5783; tau.phi[3,4] <- 5.9228;
```

```
        mu.K[3,1] <- -1.2540; mu.K[4,1] <- -1.2650;
        mu.K[3,2] <- -1.2540; mu.K[4,2] <- -1.2650;
        mu.K[3,4] <- -1.2540; mu.K[4,3] <- -1.2650;

        tau.K[3,1] <- 2.2323; tau.K[4,1] <- 2.4097;
        tau.K[3,2] <- 2.2323; tau.K[4,2] <- 2.4097;
        tau.K[3,4] <- 2.2323; tau.K[4,3] <- 2.4097;

        mu.psi[3,1] <- 0.485; mu.psi[4,1] <- 0.390;
        mu.psi[3,2] <- 0.485; mu.psi[4,2] <- 0.390;
        mu.psi[3,4] <- 0.485; mu.psi[4,3] <- 0.390;

        tau.psi[3,1] <- 7.305; tau.psi[4,1] <- 7.305;
        tau.psi[3,2] <- 7.305; tau.psi[4,2] <- 7.305;
        tau.psi[3,4] <- 7.305; tau.psi[4,3] <- 7.305;

        r.rho[3,1] <- 1.1113; r.rho[4,1] <- 1.4303;
        r.rho[3,2] <- 1.1113; r.rho[4,2] <- 1.4303;
        r.rho[3,4] <- 1.1113; r.rho[4,3] <- 1.4303;

        m.rho[3,1] <- 1.4274; m.rho[4,1] <- 1.4946;
        m.rho[3,2] <- 1.4274; m.rho[4,2] <- 1.4946;
        m.rho[3,4] <- 1.4274; m.rho[4,3] <- 1.4946;

        r.tau[1,2] <- 7.7402; r.tau[2,1] <- 8.2405;
        r.tau[1,3] <- 7.7402; r.tau[2,3] <- 8.2405;
        r.tau[1,4] <- 7.7402; r.tau[2,4] <- 8.2405;

        m.tau[1,2] <- 1.3967; m.tau[2,1] <- 1.5669;
        m.tau[1,3] <- 1.3967; m.tau[2,3] <- 1.5669;
        m.tau[1,4] <- 1.3967; m.tau[2,4] <- 1.5669;

# Data transformation
for (i in 1:n) { cz[i]<- log(CZ[i]) }

# Initial tau for SV
        tau[1,3] ~ dgamma(1,1); tau[1,4] ~ dgamma(1,1)

# Model
        for (i in 2:n) {
                mu[i,1] <- c[1]+gam[1]*log(log(tt[i]))+phi[1]*cz[i-1]
                mu[i,2] <- c[2]+b[2]*dummy[i]+cz[i-1]
                mu[i,3] <- c[3]+gam[3]*log(log(tt[i]))+phi[3]*cz[i-1]
                mu[i,4] <- c[4]+b[4]*dummy[i]+cz[i-1]

                #SV auxiliary
                for (k in 3:4) {
                        lv[i,k] <- K[k]+psi[k]*log(1/tau[i-1,k])
                        tmp[i,k] <- lv[i,k]+z[i,k]
                        tau[i,k] <- exp(-tmp[i,k])
                        z[i,k] ~ dnorm(0,rho[k]) }

        # Model probabilities and parameters - update
        cz[i]   ~       dnorm(mu[i,mod], tau[i,mod])
        }

# Model probabilities - final transformation
for (k in 1:4) {prob[k] <- step(mod-k)-step(mod-k-1)}

}

# Sample data set for model estimation and selection tasks (C.1 and C.2)

list(n=15, N=18, CZ=c(12900, 10207, 10540, 10857, 12880, 10729, 9910, 7802, 12918, 44679,
60015, 53453, 60294, 68183, 102511), dummy=c(1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), tt=c(2.7183, 2.7183, 2.7183, 2.7183,
2.7183, 2.7183, 2.7183, 2.7183, 2.7183, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 14, 13, 13,
12, 12, 11, 11, 10, 10, 9, 9, 8 )) # Trend and dummy include adjustments for definition change
```

Source for C.1 and C.2: own elaboration on the basis of the *Pines* example from the WinBUGS manual from: www.mrc-bsu.cam.ac.uk/bugs/documentation/exampVol2/node20.html (accessed on 10 June 2008), as well as its adaptation in Bijak (2008b: v-vi).

B.12. VAR models with Lindley-type Wald tests for impact of migration determinants and conditional forecasts: example of Poland

```
# From-General-To-Specific VAR modelling: Lindley-type Wald tests and conditional forecasts

model { # Three-dimensional VAR(1) model

# Priors for parameters
c[1] ~ dnorm(0,0.0001); c[2] ~ dnorm(0,0.0001); c[3] ~ dnorm(0,0.0001) # constants - diffuse

alpha[1,1] ~ dnorm(0.5,4) # Autoregressive features of migration, Question 2, Answers A & C
alpha[2,2] ~ dnorm(0,0.01); alpha[3,3] ~ dnorm(0,0.01) # own variable lags
alpha[1,2] ~ dnorm(-0.285714,4); alpha[1,3] ~ dnorm(-1.25,4) # expert based - Questions 8-9
alpha[2,1] ~ dnorm(0,1); alpha[3,1] ~ dnorm(0,1); alpha[2,3] ~ dnorm(0,1); alpha[3,2] ~ dnorm(0,1)

T[1:3,1:3] ~ dwish(P[1:3,1:3],3)     # precision matrix (Sigma^-1)
P[1,1]<-1.095686; P[1,2]<- -0.259004; P[1,3]<- -1.100765; P[2,2]<-3; P[3,3]<-3; P[2,3]<- 0
P[2,1]<- P[1,2]; P[3,1]<- P[1,3];  P[3,2]<- P[2,3] # Parameter P for the Wishart distribution

# Data definitions
for (t in 1:n) { y[t,1] <- log(VAR1[t]); y[t,2] <- VAR2[t]; y[t,3] <- VAR3[t] }
for (t in n:n+N) { x2[t] <- CVAR2[t-n+1]; x3[t] <- CVAR3[t-n+1] }

# Model
for (t in 2:n) { for (i in 1:3)
                       {
                       mu[t,i] <- alpha[i,1] * y[t-1,1] + alpha[i,2] * y[t-1,2]+alpha[i,3] *
                              * y[t-1,3] + c[i]
                       }
               y[t,1:3] ~ dmnorm(mu[t,1:3],T[1:3,1:3])
               }
for (k in 1:3) { cy.new[n,k] <- y[n,k] }

b[1] <- alpha[1,2]; b[2] <- alpha[1,3] # Vector b for the impact of variables 2 and 3 jointly

Sigma[1:3,1:3] <- inverse(T[,]) # Regression parameters for the conditional model y[t] | x[t]
beta[1] <- (Sigma[1,2] * Sigma[3,3] - Sigma[1,3] * Sigma[3,2]) / (Sigma[2,2] * Sigma[3,3] -
Sigma[2,3] * Sigma[3,2])
beta[2] <- (Sigma[1,3] * Sigma[2,2] - Sigma[1,2] * Sigma[2,3]) / (Sigma[2,2] * Sigma[3,3] -
Sigma[2,3] * Sigma[3,2])
ctau <- 1/(Sigma[1,1] - beta[1] * Sigma[2,1] - beta[2] * Sigma[3,1])

# To be run in the second round, after obtaining E{b[1]}, E{b[2]}, E{beta[1]}, and E{beta[2]}
from the initial MCMC summaries
Eb[1] <- -0.1552; Eb[2] <- 0.01553; Ebeta[1] <- -0.08308; Ebeta[2] <- -0.3662
for (i in 1:2) { for (j in 1:2) { bvar[i,j] <- b[i] * b[j] - Eb[i] * Eb[j]; betavar[i,j] <-
beta[i] * beta[j] - Ebeta[i] * Ebeta[j]} } # Variance matrices

# To be run in the third round, after obtaining D{b[1]}, D{b[2]}, D{beta[1]}, D{beta[2]} and
the covariances from the MCMC summaries
Db[1,1] <- 0.2108; Db[2,2] <- 0.003218; Db[1,2] <- 0.01233; Db[2,1] <- Db[1,2]
Dbeta[1,1] <- 0.0213; Dbeta[2,2] <- 0.02047; Dbeta[1,2] <- -1.776E-4; Dbeta[2,1] <- Dbeta[1,2]

Tb[1,1] <- Db[2,2] / (Db[1,1] * Db[2,2] - Db[2,1] * Db[1,2])
Tb[1,2] <- - Db[2,1] / (Db[1,1] * Db[2,2] - Db[2,1] * Db[1,2])
Tb[2,1] <- - Db[1,2] / (Db[1,1] * Db[2,2] - Db[2,1] * Db[1,2])
Tb[2,2] <- Db[1,1] / (Db[1,1] * Db[2,2] - Db[2,1] * Db[1,2])

Tbeta[1,1] <- Dbeta[2,2] / (Dbeta[1,1] * Dbeta[2,2] - Dbeta[2,1] * Dbeta[1,2])
Tbeta[1,2] <- - Dbeta[2,1] / (Dbeta[1,1] * Dbeta[2,2] - Dbeta[2,1] * Dbeta[1,2])
Tbeta[2,1] <- - Dbeta[1,2] / (Dbeta[1,1] * Dbeta[2,2] - Dbeta[2,1] * Dbeta[1,2])
Tbeta[2,2] <- Dbeta[1,1] / (Dbeta[1,1] * Dbeta[2,2] - Dbeta[2,1] * Dbeta[1,2])

for (i in 1:2) { ctrb[i] <- b[i] - Eb[i]; ctrbeta[i] <- beta[i] - Ebeta[i] }

HPDtest[1] <- pow(ctrb[1],2) / Db[1,1] # Impact of lagged variable 2 only
HPDtest[2] <- pow(ctrb[2],2) / Db[2,2] # Impact of lagged variable 3 only
HPDtest[3] <- ctrb[1] * (ctrb[1] * Tb[1,1] + ctrb[2] * Tb[2,1]) + ctrb[2] * (ctrb[1] * Tb[1,2]
      + ctrb[2] * Tb[2,2])                    # Impact of lagged variables 2 and 3 jointly
HPDtest[4] <- pow(ctrbeta[1],2) / Dbeta[1,1] # Conditional regression, variable 2 only
HPDtest[5] <- pow(ctrbeta[2],2) / Dbeta[2,2] # Conditional regression, variable 3 only
HPDtest[6] <- ctrbeta[1] * (ctrbeta[1] * Tbeta[1,1] + ctrbeta[2] * Tbeta[2,1]) + ctrbeta[2] *
      (ctrbeta[1] * Tbeta[1,2] + ctrbeta[2] * Tbeta[2,2])  # Conditional regression, jointly
```

```
# Conditional forecasts y[t] | x[t] = x0[t]
for (t in n+1:n+N)
	{
	cy.new[t,1:3] ~ dmnorm(cmu.new[t,1:3], T[1:3,1:3])
	mu.new[t,1] <- alpha[1,1] * cy.new[t-1,1] + alpha[1,2] * x2[t-1] + alpha[1,3] *
	* x3[t-1] + c[1]
	cmu.new[t,2] <- x2[t]; cmu.new[t,3] <- x3[t]
	}

}

# Sample data set for Lindley's tests and conditional forecasts demographic variables (C.3)

list( n = 18, N = 18,
VAR1 = c(2626, 5040, 6515, 5924, 6907, 8121, 8186, 8426, 8916, 7525, 7331, 6625, 6587, 7048,
9495, 9364, 10802, 14995), VAR2=c(0.4129, 0.3713, 0.3141, 0.2653, 0.2462, 0.1218, 0.1106,
0.0839, 0.0524, 0.0015, 0.0268, 0.013, -0.015, -0.0371, -0.0194, -0.0102, 0.012, 0.0279),
VAR3=c(64.7762, 64.8878, 65.1055, 65.3581, 65.6221, 65.9364, 66.296, 66.687, 67.18, 67.7875,
68.3826, 68.5778, 69.013, 69.4077, 69.8054, 70.1484, 70.4773, 70.7837), CVAR2=c(0.0279,
0.0091, 0.0104, 0.0091, 0.0047, -0.003, -0.0142, -0.0285, -0.0457, -0.0653, -0.0869, -0.1101,
-0.1345, -0.16, -0.1863, -0.2132, -0.2403, -0.2673, -0.2938), CVAR3=c(70.7837, 71.0558,
71.2914, 71.459, 71.5484, 71.3337, 70.9823, 70.5288, 70.0321, 69.4852, 68.8764, 68.2423,
67.6113, 67.0086, 66.4074, 65.8708, 65.3879, 64.9298, 64.5592) )
```

Source for C.3: adapted from Bijak (2008b: vii)